Studies in the Physical Foundations of Gravitational Theories

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Summary

Motivation

The General Theory of Relativity is a very successful macroscopic theory of gravity—at least to the extent that observational data is available:

> Einstein's theory lives on, as *the* macroscopic theory of gravitation. It would be awfully nice, if it were microscopic too. But it isn't. Newton successfully wrote apple = moon, but you cannot write apple = neutron.
>
> J. L. Synge

Many attempts have been made to develop a quantum theory of gravity, but all trials have failed to successfully reconcile the fundamental principles of both quantum theory and gravity.

In this dissertation, we will investigate some of the fundamental principles, the theoretical basis and the main assumptions underlying Einstein's gravitational theory in more detail. We will start with the assumptions of Einstein's special theory of relativity, and we then discuss the validity of generalizations that are usually applied when switching from Minkowski spacetime to more general spacetime manifolds. We will also compare the foundations of Einstein's theories with ideas and assumptions concerning quantum mechanics and quantum field theory.

Two main assumptions will be given special consideration: the *hypothesis of locality* and the *Einstein equivalence principle*. The hypothesis of locality assumes accelerated observers to be locally equivalent to inertial observers. This is true for uncharged point particles as well as for waves in the short-wavelength or ray approximation. We investigate the validity of this hypothesis and of alternatives, for example in the case of length measurements and in the case of electromagnetic waves.

Einstein's principle of equivalence postulates the *local* equivalence of accelerated observers and observers in a gravitational field. The locality of this postulate allows one to generalize from Minkowski spacetime to more general manifolds. We investigate the usual generalization that leads to the standard form of the general theory of relativity as well as alternative forms of such a generalization such as teleparallel theories.

Introductory Overview

We start this dissertation by introducing the foundations of the different theories that we will investigate. We present the basic assumptions, principles, and axioms of electrodynamics and the special and general theories of relativity. While we will not question most of those assumptions, our goal is to clearly delineate their domains of validity. This should help clarify the discussions in subsequent chapters; moreover, we do not want to hide any assumption, so that we and others can easily question those other assumptions not investigated in this work.

Summary

The concepts and consequences of the relativity theories are complex and not easy to understand. Some of them are contradictory to our intuition and/or experience. For this reason we outline in chapter 2 some of the basic concepts and consequences, especially if, for example, a clear working knowledge is necessary to understand various difficulties with these ideas for accelerated oberservers. We will also point out some common misconceptions about these modern theories, such as the claim that "all is relative."

In chapter 3 we discuss the hypothesis of locality and its problems in detail. We show that accelerations are connected to acceleration lengths via the speed of light. If a physical phenomenon involves a length scale, such as a wavelength, or just measuring a length, then the hypothesis of locality can lead to non-unique answers.

We then move on to discuss effects of acceleration that are based on global concepts, such as radiation of a uniformly accelerated charge. Another effect caused by acceleration is the Unruh effect, a quantum field effect that predicts that accelerated observers will see a thermal spectrum of particles. We briefly investigate these effects and their consequences in chapter 4.

In chapter 5 we discuss alternatives to the hypothesis of locality and their effects on electrodynamics as an example of a simple (Abelian) gauge theory. We will state where predictions of these theories differ, so that it is clear how—at least in principle—we can experimentally distinguish between them.

Chapter 6 deals with Einstein's principle of equivalence and the validity of generalizations that are usually applied when switching from Minkowski spacetime to more general spacetime manifolds. We will investigate the usual generalization that leads

to the standard form of the general theory of relativity as well as alternatives of such a generalization, such as the Einstein form of teleparallel theories. In chapter 7 we compare experimental results with the general form of teleparallel theories.

We finish this dissertation by drawing conclusions.

The appendices collect mathematical definitions and theorems that we use throughout this thesis for easy reference.

Notation

In this work we mainly follow the notation and conventions of [He95], except when specified otherwise. Some important rules of exterior calculus are collected in appendix B.

Let us summarize some conventions of [He95] here:

Indices

Anholonomic indices are represented by Greek letters from the beginning of the alphabet, e.g. α, β, γ,.... In four dimensions (e.g. spacetime) they count from $\hat{0}$ to $\hat{3}$. Letters from the middle of the Latin alphabet, e.g. i, j, k,..., are used for holonomic indices, counting from 0 to 3 for four-dimensional manifolds.

We use the summation convention: We automatically sum over the same upper and lower index. For these so-called *dead indices* we will use letters from the beginning as well as the middle of the Greek alphabet.

Square brackets $A_{\dots[\alpha\beta]\dots}$ imply the antisymmetrization of the included indices, while parentheses $A_{\dots(\alpha\beta)\dots}$ mean symmetrization of indices. The exclusion of indices from the symmetrization or antisymmetrization is indicated by two vertical lines around them: $A_{\ldots[\alpha|\gamma\ldots|\beta]\ldots}$.

Symbols

The symbol ":=" is used to define a symbol or abbreviation written on the side of the colon by the expression on the other side.

If a variable is set equal to a constant fixed value, the symbol "$\equiv$" is used, e.g. $f(x) \equiv 1$.

If an equation is only valid in a specific reference frame with respect to fixed tetrads or a fixed metric we will indicate this "equality with respect to specific tetrads" by the symbol "$\stackrel{*}{=}$".

Local Minkowski Metric

We choose

$$o_{\alpha\beta} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} =: \mathrm{diag}(+1, -1, -1, -1)$$

for the local Minkowski metric, unless explicitly specified otherwise. The *index of a metric* (cf. appendix B.5) for four-dimensional spacetime manifolds is therefore 3.

Units

The fundamental constants of nature such as the speed of light c, Newton's gravitational constant G, and Planck's constant $\hbar$ can be combined to get numbers of dimension energy, mass, time, and length. These numbers are called Planck energy, Planck mass, Planck time, and Planck length respectively. In SI units the respective values are[1]:

$$
\begin{aligned}
E_{\text{Planck}} &= \sqrt{\frac{\hbar c^5}{G}} = 1.96 \cdot 10^{9}\,\text{J} = 1.22 \cdot 10^{28}\,\text{eV} \;, \\
m_{\text{Planck}} &= \sqrt{\frac{\hbar c}{G}} = 21.8\,\mu\text{g} \;, \\
t_{\text{Planck}} &= \sqrt{\frac{\hbar G}{c^5}} = 5.4 \cdot 10^{-44}\,\text{s} \;, \\
l_{\text{Planck}} &= \sqrt{\frac{\hbar G}{c^3}} = 1.6 \cdot 10^{-35}\,\text{m} \;.
\end{aligned}
$$

Setting one or two of the constants of nature to unity yields a simpler, although physically less intuitive, unit system, e.g. *geometrical units* or *natural units*.

For *geometrical units* we set $c = 1$ and $G = 1$ [ON83]. This gives the following relations between dimensions:

$$
\begin{array}{ccccccccc}
[\text{energy}] & = & [\text{mass}] & = & [\text{time}] & = & [\text{length}] & & \\
E_{\text{Planck}} & = & m_{\text{Planck}} & = & t_{\text{Planck}} & = & l_{\text{Planck}} & = & \sqrt{\hbar} \;.
\end{array}
$$

On the other hand, for *natural units* (also called *fundamental units*) we set $c = 1$ and $\hbar = 1$. Now this yields the following relationships between dimensions:

$$
\begin{array}{ccccccccc}
[\text{energy}] & = & [\text{mass}] & = & [\text{time}]^{-1} & = & [\text{length}]^{-1} & & \\
E_{\text{Planck}} & = & m_{\text{Planck}} & = & t^{-1}_{\text{Planck}} & = & l^{-1}_{\text{Planck}} & = & {}^{1}\!/\!\sqrt{G} \;.
\end{array}
$$

[1] If one chooses Einstein's gravitational constant $\kappa := \frac{8\pi}{c^4}G$ instead of Newton's gravitational constant G, the values of the Planck constants will be different by factors of $\sqrt{8\pi}/c^2$ or $c^2/\sqrt{8\pi}$.

Contents

1 Introduction

We first want to present the basic assumptions, principles, and axioms for the structure of spacetime, for electrodynamics, and the special and the general theories of relativity. While we will not question most of these assumptions, our goal is to clearly outline them in this chapter. This outline should make it possible for future work to clearly know which assumptions could be questioned in the future and which consequences will still be valid or fall with a change of those assumptions.

1.1 Structure of Spacetime

We assume that spacetime is a sufficiently 'regular' 4-dimensional continuum, i.e. we model spacetime as a 4-dimensional connected differentiable manifold, that is Hausdorff, orientable, and paracompact. We do *not* assume a metric or a connection. Furthermore, we demand that the manifold can be split into $1+3$ dimensions, i.e., into time and space. Additionally we assume that, at each point of spacetime, there exists a vector frame of reference or vector basis field e_α, with $\alpha = \hat{0}, \hat{1}, \hat{2}, \hat{3}$, see appendix B.2.

As a special case, we define *Minkowski spacetime* to be an affine space, see appendix A.1, with a Minkowski metric $o_{\alpha\beta} = \mathrm{diag}(+1, -1, -1, -1)$.

1.2 Electrodynamics

For the structure of electrodynamics we follow the axioms of Charge & Flux electrodynamics, as presented by F. W. Hehl and Yu. N. Obukhov in [He99].

Axiom 1 on *electric charge conservation*: We assume that to each arbitrary 3-dimensional volume in space we can attach a number, called *electric charge*, which can be positive, zero, or negative. This charge is assumed to be conserved. Mathematically, this translates into a 4-dimensional electric current J which is an odd 3-form that fulfills $\oint J = 0$ for arbitrary closed 3-cycles. As a consequence of a theorem of de Rham, we find $dJ = 0$ and $J = dH$, i.e., the current has the excitation H, which is an odd 2-form, as a potential.

Decomposed into 1+3, the current reads $J = (\rho, j)$, the excitation $H = (\mathcal{D}, \mathcal{H})$, and the exterior derivative $d = \left(\dot{()}, \underline{d}\right)$. The electric excitation $\mathcal{D}$ is conventionally called *dielectric displacement*, the magnetic excitation $\mathcal{H}$ *magnetic field*. Both quantities are more than potentials since they have their own operational interpretation using Maxwellian double plates and the Gauss compensation method, respectively.

As a consequence of the $1 + 3$ decomposition, $dJ = 0$ turns out to be what is traditionally known as *continuity equation for electric charge*

$$\underline{d}\, j + \dot{\rho} = 0 \, , \tag{1.1}$$

whereas $J = dH$ translates into the inhomogeneous Maxwell equations:

$$\rho = \underline{d}\,\mathcal{D}\,, \qquad j = \underline{d}\,\mathcal{H} - \dot{\mathcal{D}}\,. \tag{1.2}$$

Therefore, the inhomogeneous Maxwell equations are a consequence of charge conservation. (Usually, in textbooks, one finds the reverse statement, namely that charge conservation is a consequence of the inhomogeneous Maxwell equations.) The (active) electromagnetic excitation $H = (\mathcal{D}, \mathcal{H})$ is caused by the source $J = (\rho, j)$ and is an additive or extensive quantity.

Axiom 2 on the *Lorentz force density*: We assume that we can prepare a test charge and measure the corresponding Lorentz force. Since the electric current J is known from axiom 1 and the notion of force from mechanics, we can define the *electromagnetic field strength* $F = (E, B)$ as an even 2-form in the standard way, with the electric field E and the magnetic field B, using the assumption of the Lorentz force density as an odd covector-valued 4-form, $f_\alpha = (e_\alpha \rfloor F) \wedge J$. The (passive) electromagnetic field strength $F = (E, B)$ is an intensity. The essential space component of the Lorentz force density reads ($a = 1, 2, 3$):

$$f_a = \rho \wedge (e_a \rfloor E) + j \wedge (e_a \rfloor B)\ . \tag{1.3}$$

Axiom 3 on *magnetic flux conservation*: By means of F, we can define *magnetic flux* and can attach to each arbitrary 2-dimensional surface in spacetime a scalar number. We postulate conservation of magnetic flux $\oint F = 0$ for arbitrary closed 2-cycles. This yields $dF = 0$ and $F = dA$.

Again we 1 + 3 decompose and find, from $dF = 0$, the homogeneous Maxwell equations,

$$\underline{d}\, B = 0\,, \qquad \underline{d}\, E + \dot{B} = 0\,, \tag{1.4}$$

and from $F = dA$ the determining equations for the electromagnetic potentials,

$$B = \underline{d}\, A\,, \qquad E = -\underline{d}\, \varphi + \dot{A}\,. \tag{1.5}$$

Axiom 4 on the constitutive law: We postulate a link between the odd excitation 2-form $H = H_{ij}\, dx^i \wedge dx^j/2$ and the even field strength 2-form $F = F_{ij}\, dx^i \wedge dx^j/2$. Here the structure of spacetime (its *constitution*) enters. Usually a specific *linear* law $H \sim {}^\star F$, with the odd Hodge star operator *, see appendix B.6, is assumed for vacuum. But nonlocal and nonlinear laws are also possible, as we will discuss in later chapters. If we postulate an arbitrary linear law

$$H_{ij} = \frac{1}{2}\, \kappa_{ij}{}^{kl} F_{kl} \tag{1.6}$$

then a metric of spacetime can be *derived* therefrom (up to a conformal factor), see [He99].

In the spirit of our axiomatics, charge conservation is analogous to flux conservation; therefore, there is no natural place for magnetic charge in our formalism. Similarly, the inhomogeneous Maxwell equations are analogous to the formulas (1.5) for the potentials.

1.3 Einstein's special theory of relativity

Einstein's special theory of relativity (SR) describes physics in a 4-dimensional Minkowski spacetime. A *standard observer* is an observer that is at rest in an inertial frame together with devices that measure physical quantities. One of the main principles that the special theory of relativity is based on is the *Poincaré invariance of physical measurements in inertial frames* (invariance with respect to Poincaré transformations, i.e. inhomogeneous Lorentz transformations). This means that all standard observers (*unaccelerated* observers in an inertial frame) will measure the same physics. Numerical results just differ because the observers happen to choose different coordinate systems for their inertial frames. After applying Poincaré transformations between these different coordinate systems, the results will coincide with each other and all observers agree about the physical event.

This assumption includes the hypothesis that physics doesn't change if passive displacements occur in space or time. These concepts are called *homogeneity of space* (translations in space), *homogeneity of time*, and *isotropy of space* (rotations in space). These assumptions are currently not directly testable and might very well not be possible to ascertain in principle. However, they are the most reasonable assumptions one can start with. Indirect evidence from space observations is consistent with these assumptions.

The Poincaré transformations become Galilei transformations, if the speed of light c reaches infinity. However, experiments show c to be constant and of a finite value. According to our assumption of Poincaré invariance, this value is the same in all inertial frames—however, we don't claim anything about the value of c in accelerated frames. The assumption of the existence of a finite limiting speed is sometimes called the second postulate of SR; however, it follows from the Lorentz invariance of Maxwell's equations.

For some purposes we will restrict ourselves to Lorentz transformations between inertial frames which are the homogeneous Poincaré transformations (boosts and rotations).

1.4 Einstein's general theory of relativity

Einstein's general theory of relativity (GR) is a macroscopic theory of gravity. GR generalizes from Minkowski spacetime to arbitrary spacetime manifolds and explains gravitational effects in a purely geometrical manner.

Einstein's theory of gravity is based on the *principle of equivalence*. It postulates the *local* equivalence between an observer in a gravitational field and an accelerated observer in Minkowski spacetime, see chapter 6, i.e. both observers are postulated to measure the same physics locally. As we will discuss in detail later, we have to apply an additional assumption to know what accelerated observers actually measure before we can use the equivalence principle. In fact, this additional assumption is the Hypothesis of Locality; namely, that an accelerated observer measures the same physical results as an inertial observer with the same position and velocity.

When we generalize from the affine Minkowski spacetime to a spacetime manifold, the following question arises: How should we connect the local neighborhoods of the manifold (or rather: how should we connect the affine tangent spaces, which are the local representations of Minkowski spacetime, see appendix A.5)? The traditional choice is to introduce a metric g_{ij} and a Christoffel connection $\overset{\{\}}{\Gamma}$ as derivative of this metric:

$$\overset{\{\}}{\Gamma}_{ij}{}^{k} := \frac{1}{2}\, g^{kl} \left(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij}\right) \ . \tag{1.7}$$

This Christoffel connection also arises naturally when transforming from a global Minkowski coordinate system to an accelerated coordinate system. In this case, however, the Christoffel connection fulfills an integrability condition, namely, that its Riemann-Christoffel curvature is vanishing. In the general case of connecting local neighborhoods of a manifold, this integrability condition is removed, which in general creates a pseudo-Riemannian spacetime with non-vanishing curvature and yields the famous Einstein equations:

$$G_{ij} := \mathrm{Ric}_{ij} - \frac{1}{2}\, g_{ij} R = \frac{8\pi G}{c^4}\, T_{ij} \ , \tag{1.8}$$

with T_{ij} the energy-momentum tensor.

But this is not the only possibility. With a different choice one could remove a different integrability condition (thereby keeping the spacetime curvature-free). We will discuss this further in more detail in chapter 6.

1.5 Basic Assumptions

The formalism of the described theories is widely accepted and highly successful: All known experiments coincide within their accuracy with the predictions of the theories. Unfortunately, there are differences in interpreting these theories. Different textbook authors base their explanations on different assumptions.

In order to understand the physical structure of the presently known models of nature it should be helpful to reinvestigate the basic assumptions of GR. All scientific investigations are based on fundamental principles; it is therefore essential to clearly know and show these principles, and to investigate alternatives to the common assumptions that sometimes are hidden in the presentations. If we just tacitly assume basic foundations we might miss better alternative theories that might bring us a step closer to unifying gravity with quantum theory.

We will base our discussions of basic assumptions on the question: "What do or can observers measure?" Some old and new assumptions will remain untestable because physical principles forbid a measurement, others will remain untestable because of the accuracy of contemporary equipment. However, we will of course pay close attention that our alternative assumptions are consistent with currently known experimental results.

Quantum mechanics

Microscopic processes are described by quantum mechanics. While light often portrays wave properties in macroscopic experiments, there are microscopic as well as macroscopic experiments where light can best be understood as a particle. A similar particle-wave duality can be introduced for all particles and wave phenomena. We can call this the *complementarity principle*.

Some traditional assumptions that we will investigate in later chapters will only be valid for uncharged point particles and for waves in the short-wavelength or ray approximation. These assumptions, therefore, cannot be brought into harmony with quantum theory without further modifications.

2 Measurement Theory: Concepts and Consequences of SR and GR

In this chapter we first discuss how local we can really get when we measure something. This is an interesting question since it is usually assumed that fields are defined at pointlike events and that our assumptions are true in a sufficiently small neighborhood.

The concepts and consequences of the theories of modern physics, especially those of SR and GR, are complex and not easy to understand. Some of them are contradictory to our intuition and/or experience. We will discuss the most important statements of SR for inertial frames, so that we can investigate differences in accelerated frames later on.

2.1 Measurements and their relation to the assumptions

As outlined in the last chapter, one of the main axioms of SR is the claim that all inertial observers will measure the same physics, only expressed in different coordinate systems. This axiom led to the increased usage of differential forms in SR and GR: differential forms describe the physics of a phenomenon in a coordinate-independent manner.

If this axiom is supposed to make sense, we have to ask the question what "measuring physics" actually means. First, we have to make clear, which physical quantities can be and are defined operationally. For example, the definition of the electromagnetic field strength $F = (E, B)$ in our Axiom 2

$$f_\alpha = (e_\alpha \rfloor F) \wedge J = \rho \wedge (e_\alpha \rfloor E) + j \wedge (e_\alpha \rfloor B) \tag{2.1}$$

is based on the already defined notions of force and current. Since the force on a charged test particle can be measured, we can determine the field strength at the position of the test particle.

The test particle will have a finite size, though, so the "position of the test particle" might be hard to determine. Since a measuring device of point-particle size doesn't exist, all classical measurements of, e. g. $E(t, x)$, include time and volume averages

$$< E >= \frac{\int E(t, x')\, dt\, \overbrace{d^3x'}^{=dV}}{\Delta t\, \Delta V} \tag{2.2}$$

in *real* measurements [BR33, BR50]. In curvature-free and torsion-free spaces like the affine spacetime of SR we expect that these averages work out to give reasonable local results for smooth functions. However, if the geometry allows to define at least one length scale, it is not so clear anymore that the averaging procedure is still viable in producing local fields.

This is a problem that already exists in classical physics. Quantum mechanics introduces new possible complications. The commutation rule

$$[x, p] = i\hbar \tag{2.3}$$

has consequences for the equations that operationally define new physical quantities. These issues have been discussed at length by Bohr and Rosenfeld [BR33, BR50].

2.2 Special relativity – simultaneity and length measurements

An *event* in SR is associated with a *single* location in space and a *single* instant in time. The *position of an event* is defined to be the coordinate label on a rigid ruler that extends from the spatial origin to the event; this notion is then naturally extended to the spatial coordinates that characterize the location of the event in space. The ruler is envisioned to extend indefinitely from some chosen origin. Such a choice is only possible in a global inertial coordinate frame, which can be defined only in Minkowski spacetime for inertial observers. The *time of an event* is most naturally defined as the reading on a clock located at the event's position at the instant at which the event occurs. The rulers and clocks used by an inertial observer are at rest relative to the observer. Time is somehow a difficult notion to grasp, especially when it becomes frame dependent under Lorentz transformations. For an investigation of student understanding after instruction, demonstrating such difficulties in grasping these and other concepts, see [Sche00].

Simultaneity

All inertial observers in SR are assumed to be either actual *intelligent observers* or measuring devices that use synchronized clocks. To determine the time of a distant event, an observer corrects for the travel time of a signal originating at the event. To perform this correction the observer has to know the distance to the event by either determining the event's spatial coordinates in its reference frame or by prior measurement of the distance. The determination of the location and the time of an event are independent of the position of an observer compared to all other observers in the same reference frame.

The time ordering of the events depends on the relative velocity of the inertial observers and the relative position of the events, but not the positions of the observers since global synchronization of clocks is assumed. The invariance of the speed of light c has an additional immediate implication: Two events at different locations that occur at the same time in a given inertial frame are not simultaneous in any other inertial frame. Moreover, $v < c$ for any observer implies that the causal sequence of events is independent of the inertial observers.

Length measurements

An inertial frame is globally defined, since the lifetime of clocks can be ideally extended indefinitely and the rulers ideally extend indefinitely in space. Hence, lengths are simply determined by the differences of the coordinate positions of the endpoint of line segments at the same time in such a reference frame, i.e. $L = |\vec{x}_2 - \vec{x}_1|$ is the length of the straight line segment extending from $\vec{x}_1$ to $\vec{x}_2$. In effect, the homogeneity and isotropy of spacetime in an inertial frame allows us to sum intervals of time and space corresponding to the use of finite clocks and rulers.

A ruler of length l_0 at rest in an inertial frame contracts by a factor of

$$\gamma^{-1} = \sqrt{1 - \frac{v^2}{c^2}} \tag{2.4}$$

as measured by standard observers at rest in an inertial frame moving with speed $v = \beta c$ along the direction defined by the ruler; this effect is known as the Lorentz-Fitzgerald contraction. A related effect is time dilation; this can be described in terms of the relationship between the proper spacetime interval $\Delta\tau$ and the coordinate time interval Δt, namely, $\Delta\tau = \Delta t\sqrt{1 - \frac{v^2}{c^2}}$.

As we will discuss later, accelerated observers cannot use globally-defined rest frames; accelerated frames are only valid within certain neighborhoods. Therefore, we cannot use coordinate positions to determine distances. Since the *prior* knowledge of distances is necessary for the synchronization of clocks for observers at different positions, we only can use a passive second observer in an operational definition of distance.

It is possible to implement an operational definition of distance between two inertial observers using electromagnetic signals: One observer at rest at $\vec{x}_1$ in some inertial frame sends out a light signal towards a second (possibly moving) observer. The second observer at $\vec{x}_2$ sends a light signal back immediately after reception of the first light signal. The first observer determines the time difference Δt between sending the first light signal out and receiving the second light signal at its position. The length between the observers is then given by

$$L^* := \frac{1}{2}\, c\, \Delta t \; . \tag{2.5}$$

This length definition relies only upon the assumption that the speed of light is constant and equal to c in all inertial reference frames; moreover it is consistent with the measurement of length based on rulers (i.e. $L^* = L$).

2.3 Special relativity – accelerated observers

In Minkowski spacetime, one may consider inertial observers as well as accelerated observers. The fundamental laws of physics (classical and quantum) have been formulated with respect to inertial observers. Before we discuss accelerated observers, we consider the measurement of acceleration by inertial observers.

Acceleration is absolute

Common knowledge often claims that Einstein's theories say that "*all* is relative." This is not true: For example, every observer can agree if an object is accelerated or not. Since this is essential for our discussion, let us quickly remind ourselves of this fact. With $x^\mu(\tau)$ as the path of the object in arbitrary, admissible coordinates in

Minkowski spacetime, we have $v^\mu = \frac{dx^\mu}{d\tau}$ as its 4-velocity. We then have the equation of motion

$$A^\mu(\tau) := \frac{Dv^\mu}{D\tau} = \frac{d^2x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}{}^\mu \frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} \tag{2.6}$$

where $\Gamma_{\alpha\beta}{}^\gamma$ are the Minkowskian Christoffel-symbols (with zero curvature). For the 4-velocity we have $v^\mu v_\mu = c^2$ (a constant), therefore we know

$$\frac{Dv^\mu}{D\tau} v_\mu = A^\mu v_\mu = 0\ . \tag{2.7}$$

Since v^μ is timelike, A^μ must be spacelike, which means that we can write $A^\mu A_\mu = -g^2(\tau)$. If $g(\tau) = 0$, then A^μ must be 0 (it cannot be a null vector, since it is spacelike), so we have a free, unaccelerated particle in any coordinate system. If $g(\tau) \neq 0$, then A^μ cannot be zero in any coordinate system. So, every observer agrees if an object or another observer is accelerated or not.

Translational and rotational accelerations

An inertial observer is an ideal that cannot be realized in practice. All actual observers are accelerated. To develop the theory of accelerated systems, let us define an orthonormal frame field e_α for an accelerated observer. The components of the frame field are $\lambda^\mu{}_{(\alpha)} := e^\mu{}_\alpha$, where $e_\alpha = e^\mu{}_\alpha \partial_\mu$. We choose e_0 to be the unit vector $u^\mu(\tau) := \frac{1}{c}\frac{dx^\mu}{d\tau}$ that is tangent to the worldline at a given event $x^\mu(\tau)$ and we parametrize the remaining frame vectors characterizing the spatial directions also by τ, which is a temporal parameter measured along the accelerated path by *the standard (static inertial) observers in the underlying global inertial frame according to the formula* $\tau = \int \sqrt{1 - \frac{v^2(t)}{c^2}}\, dt$.

The condition of orthonormality for the frame field reads

$$g_{\mu\nu}\lambda^{\mu}{}_{(\alpha)}(\tau)\,\lambda^{\nu}{}_{(\beta)}(\tau) = o_{\alpha\beta} = \mathrm{diag}(+1,-1,-1,-1)\ . \tag{2.8}$$

The covariant derivative of the frame field can be expressed in the frame basis:

$$\frac{D\lambda^{\mu}{}_{(\alpha)}}{D\tau} = \Phi_{\alpha}{}^{\beta}(\tau)\lambda^{\mu}{}_{(\beta)}\ . \tag{2.9}$$

Using the orthonormality condition and the assumption of vanishing non-metricity (that means: $Dg_{\mu\nu} = 0$), we find that $\Phi_{\alpha\beta}$ is antisymmetric

$$\Phi_{\alpha\beta}(\tau) = -\Phi_{\beta\alpha}(\tau)\ ; \tag{2.10}$$

we therefore define

$$\Phi_{\alpha\beta} := \left[\begin{array}{c|c} 0 & \vec{a}/c \\ \hline -\vec{a}/c & \vec{\Omega} \end{array}\right]\ , \tag{2.11}$$

where $\Phi_{0i} = a_i/c$ and $\Phi_{ij} = \varepsilon_{ijk}\Omega^k$. Here $\vec{a}$ represents the "electric" component and is the translational acceleration, while $\vec{\Omega}$ represents the "magnetic" component and is the rotational frequency of the local spatial frame (with respect to the local nonrotating, i.e. Fermi-Walker transported, axes).

Let us restrict ourselves to static inertial coordinates x^{μ} in Minkowski spacetime for the following. We now introduce a geodesic coordinate system X^{μ} in the neighborhood of the accelerated path. At any time τ along the accelerated worldline (see figure 2.1),

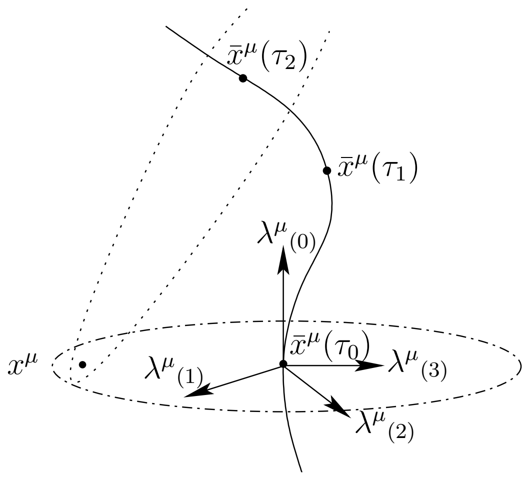

FIGURE 2.1: An event x^μ as seen by the observer $\bar{x}^\mu(\tau_0)$ with its frame field $\lambda^\mu{}_{(\alpha)}$. The geodesic coordinate system $X^\mu = (c\tau, \vec{X})$ is limited in space: If we go beyond the time τ_1, for example, coordinate assignments would start to overlap, as shown for the time τ_2. Since this cannot be accepted, spatial coordinates have to be limited in general. Thus the geodesic coordinate system is in general valid in a sufficiently narrow worldtube along the timelike worldline of the observer.

the hypersurface orthogonal to the worldline is Euclidean space and one can describe some event on this hypersurface at x^μ to be at X^μ, where x^μ and X^μ are connected via $X^0 = c\tau$ and

$$x^\mu = \bar{x}^\mu(\tau) + X^i \lambda^\mu{}_{(i)}(\tau) \tag{2.12}$$

with $i = 1, 2, 3$ and where $\bar{x}^\mu$ represents the position of the accelerated observer.[1]

From (2.12) we can derive (compare also with [He90b] and references therein) the relation

$$\begin{aligned}
dx^\mu &= \frac{1}{c}\frac{d\bar{x}^\mu}{d\tau} dX^0 + dX^i \lambda^\mu{}_{(i)} + X^i d\lambda^\mu{}_{(i)} \\
&= \lambda^\mu{}_{(0)} dX^0 + dX^i \lambda^\mu{}_{(i)} + \frac{1}{c} X^i \, dX^0 \Big[\overbrace{\Phi_i{}^0}^{=a_i/c} \lambda^\mu{}_{(0)} + \overbrace{\Phi_i{}^j}^{=\varepsilon_{ijk}\Omega^k} \lambda^\mu{}_{(j)} \Big] \\
&= \left[\left(1 + \frac{\vec{a} \cdot \vec{X}}{c^2} \right) \lambda^\mu{}_{(0)} + \frac{1}{c} \left(\vec{\Omega} \times \vec{X} \right)^i \lambda^\mu{}_{(i)} \right] dX^0 + \lambda^\mu{}_{(i)} \, dX^i \, ,
\end{aligned} \tag{2.13}$$

and hence the metric is

$$\begin{aligned}
ds^2 &= o_{\mu\nu} \, dx^\mu \, dx^\nu \\
&= \left[\left(1 + \frac{\vec{a} \cdot \vec{X}}{c^2} \right)^2 - \left(\frac{\vec{\Omega} \times \vec{X}}{c} \right)^2 \right] (dX^0)^2 - 2 \left(\frac{\vec{\Omega} \times \vec{X}}{c} \right) \cdot d\vec{X} dX^0 - \delta_{ij} \, dX^i \, dX^j \, .
\end{aligned} \tag{2.14}$$

Since we started from a global inertial frame in Minkowski spacetime, the spatial part of the line element yields Euclidean space with its origin occupied by the accelerated

[1] The new coordinate system is built only on the position of the observer and the basis field. With this assumption, we use a mathematical version of the Hypothesis of Locality. Implications for physics will be discussed in detail later.

observer.

This set of coordinates is limited. If we follow the above procedure for two different times of the accelerated observer, our new coordinates may not be unique, see figure 2.1. Since we cannot accept two sets of coordinates in the same system for one event, we have to require that the laboratory be sufficiently small. The charts for our coordinates cannot be global for accelerated observers. In fact, such geodesic coordinates are admissible as long as

$$\left(1+\frac{\vec{a}\cdot\vec{X}}{c^2}\right)^2 > \frac{1}{c^2}\left(\vec{\Omega}\times\vec{X}\right)^2 . \tag{2.15}$$

Thus in the discussion of the admissibility of the geodesic coordinates, two independent acceleration lengths must be considered: the translational acceleration length c^2/a and the rotational acceleration length c/Ω that appear in equation (2.15).

The acceleration radii are connected with the domain of applicability of the geodesic coordinate system around the reference accelerated observer. It turns out that these acceleration lengths have another independent and much more fundamental significance in terms of the *local* measurements of the accelerated observer following the reference trajectory [Mas89, Mas90]. This basic issue is discussed in chapter 3.

It is important to remark here that one may use other (more complicated) accelerated coordinate systems; however, these have their attendant difficulties [Mar96]. A discussion of these problems is beyond the scope of this thesis; therefore, we limit our considerations here to geodesic coordinate systems.

Length scales for accelerated observers

The translational and rotational "accelerations" a_i and Ω^k depend in general on both the velocity and the acceleration of the observer. We therefore construct the scalar invariants of the antisymmetric tensor $\Phi_{\alpha\beta}$, which are then independent of the (coordinate-dependent) velocity:

$$\begin{aligned} I &= \frac{1}{2c^2}\Phi_{\alpha\beta}\Phi^{\alpha\beta} = -\frac{a^2}{c^4} + \frac{\Omega^2}{c^2} \,, \\ I^* &= \frac{1}{4c^2}\Phi^*_{\alpha\beta}\Phi^{\alpha\beta} = -\frac{\vec{a}}{c^2}\cdot\frac{\vec{\Omega}}{c} \,, \end{aligned} \tag{2.16}$$

where $\Phi^*_{\alpha\beta}$ is the dual of $\Phi_{\alpha\beta}$, i. e. $\Phi^*_{\alpha\beta} = \varepsilon_{\alpha\beta\gamma\delta}\Phi^{\gamma\delta}$. We define the finite lengths $|I|^{-1/2}$ and $|I^*|^{-1/2}$ as the *proper acceleration lengths*.

Let us now see how long these lengths are in typical situations on the earth. For the translational acceleration length on the earth's surface we get ($a = 9.8\,\mathrm{m/s^2}, \Omega = 0$)

$$\frac{c^2}{a} = \frac{(3\cdot 10^8\,\mathrm{m/s})^2}{9.8\,\mathrm{m/s^2}} = 9.46\cdot 10^{15}\,\mathrm{m} \approx 1\,\mathrm{ly} \,, \tag{2.17}$$

and for the rotational acceleration ($a = 0, \Omega = \Omega_\oplus$) the result is

$$\frac{c}{\Omega} = \frac{3\cdot 10^8\,\mathrm{m/s}}{7.272\cdot 10^{-5}\,\mathrm{s}^{-1}} = 4.1253\cdot 10^{12}\,\mathrm{m} \approx 27.5\,\mathrm{AU} \,. \tag{2.18}$$

If we take a wavelength for a typical optics experiment, $\lambda \approx 10^{-7}\,\mathrm{m}$, the factor $\lambda/\mathcal{L}$ is around 10^{-23}. As long as all length scales are very small compared to the acceleration lengths, it seems reasonable to assume that differences between observations by accelerated and comoving inertial observers will also be very small.

2.4 Gravitoelectromagnetism

According to the principle of equivalence, similar length scales have to emerge in gravity. We want to estimate the order of magnitude of such gravitational lengths and compare them with length scales of typical experiments.

We use the linear approximation of general relativity for small perturbations of the Minkowski metric $o_{\mu\nu}$:

$$g_{\mu\nu} = o_{\mu\nu} + h_{\mu\nu} \ . \tag{2.19}$$

The trace-reversed perturbation metric

$$\bar{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2} o_{\mu\nu} h_{\kappa}{}^{\kappa} \tag{2.20}$$

then obeys the wave equation

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \ , \tag{2.21}$$

where the Lorentz gauge condition $\partial_\nu \bar{h}^{\mu\nu} = 0$ has been imposed.

We consider "nonrelativistic" sources that give $\bar{h}_{00} = 4\Phi^g/c^2$, $\bar{h}_{0i} = -2A_i^g/c^2$ and $\bar{h}_{ij} = \mathcal{O}(c^{-4})$, where Φ^g and $\vec{A}^g$ can be interpreted as the gravitoelectric and gravitomagnetic potentials respectively (see, e.g., [Mas99b]). We can then write the full space-time metric as

$$ds^2 = \left(1 - \frac{2\Phi^g}{c^2}\right) (dx^0)^2 + \frac{4}{c^2}\left(\vec{A}^g \cdot d\vec{x}\right) dx^0 - \left(1 + \frac{2\Phi^g}{c^2}\right) \delta_{ij}\, dx^i\, dx^j \ . \tag{2.22}$$

When we expand the squares in (2.14) and ignore squared terms and when we also ignore the gravitoelectric potential in the purely spatial part g_{ij} of (2.22) (after all, it's a linearization and the deviation from the Euclidean space remains uninterpreted in the gravito-electromagnetic approach), then we can identify

$$\Phi^g \sim -\vec{a}\cdot\vec{X} \quad \text{and} \qquad \vec{A}^g \sim -\frac{c}{2}\vec{\Omega}\times\vec{X}\ . \tag{2.23}$$

From this we see that for constant $\vec{a}$ and $\vec{\Omega}$, in agreement with what would be expected from Einstein's principle of equivalence,

$$\vec{a} = \vec{E}^g = -\vec{\nabla}\Phi^g \quad \text{and} \qquad -\vec{\Omega} = \frac{1}{c}\vec{B}^g = \frac{1}{c}\vec{\nabla}\times\vec{A}^g\ . \tag{2.24}$$

With

$$\Phi^g = \frac{GM}{R} \quad \text{and} \qquad \vec{A}^g = \frac{G}{c}\frac{\vec{J}\times\vec{R}}{R^3}\ , \tag{2.25}$$

we can approximate (on the surface of the earth)

$$\frac{c^2}{a} = \frac{c^2}{\frac{GM}{R^2}} \approx 1\,\mathrm{ly} \tag{2.26}$$

for the gravitoelectric length scale and

$$\begin{aligned}\frac{c}{\Omega} \approx \frac{c}{\frac{GJ}{c^2R^3}} \approx \frac{c}{\frac{\frac{2}{5}MR_0^2\Omega_\oplus G}{c^2R^3}} = \frac{5}{2}\frac{c}{\Omega_\oplus}\left(\frac{GM}{c^2R_0}\right)^{-1}\left(\frac{R}{R_0}\right)^3 \approx 1.44\cdot 10^9\cdot 27.5\,\mathrm{AU}\cdot\left(\frac{R}{R_0}\right)^3 \\ \approx 6\cdot 10^{21}\,\mathrm{m}\cdot\left(\frac{R}{R_0}\right)^3\end{aligned} \tag{2.27}$$

for the gravitomagnetic length scale. Again, all experimental length scales are tiny in relation to these huge gravitational length scales.

Quantum-mechanical length scales

A fundamental assumption in quantum mechanics is that the measuring devices are classical in nature and hence the Compton wavelength associated with any measuring device should be negligible compared to its dimension D [SW58]. Therefore, the dimensions of the device must obey

$$\frac{\hbar}{Mc} \ll D \, . \tag{2.28}$$

When we combine this condition with our proper acceleration length ($D \ll \mathcal{L}$), we get the conditions

$$a \ll \frac{Mc^3}{\hbar} \quad \text{and} \qquad \Omega \ll \frac{Mc^2}{\hbar} \, . \tag{2.29}$$

Therefore, all classical systems cannot have accelerations or rotations above these *maximal proper accelerations* [Ca92, FLPS97, LPS07].

3 The Hypothesis of Locality and its Problems

In the last section we saw that accelerated observers cannot define a global coordinate system anymore. Hence, their notion of simultaneity and of distances has to be reconsidered.

Different inertial observers and their reference frames are connected by Poincaré transformations which depend on the relative velocity between these observers. If we want to expand coordinate transformations to arbitrary observers, we have to investigate if that transformation can solely depend on the relative velocity between observers at a certain instant or if this transformation also should depend on the acceleration of the observers.

This question gains relevance if one considers that standard inertial observers do not really exist. All observers in reality are usually somehow accelerated, e.g. because of the earth's rotation.

In this chapter we will first present the hypothesis of locality, which claims that transformations to accelerated coordinate frames should be *locally* independent of the acceleration. We then investigate consequences of this important assumption.

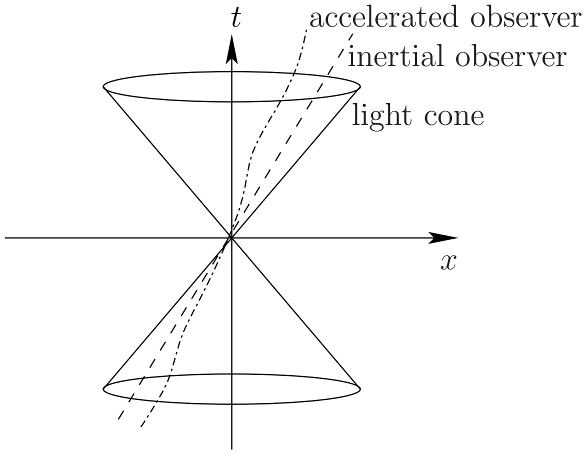

FIGURE 3.1: An inertial observer has a straight path in a spacetime-diagram, while the path of an accelerated observer is (at least somewhere) curved during its acceleration.

3.1 The Hypothesis of Locality

In a spacetime diagram an inertial observer can be portrayed as a straight line. An observer that is linearly accelerated at some time will have a curved path at this time, see figure 3.1. What will this accelerated observer measure? Typically, the *Hypothesis of Locality* [Mas89, Mas90] is tacitly assumed:

> An accelerated observer measures the same physical results as a standard inertial observer that has the same position and velocity at the time of measurement.

The curved path of the observer is substituted by the straight line tangential to the curve at the time of measurement. The radius of curvature of the accelerated worldline is characterized by the acceleration length $\mathcal{L}$; the hypothesis of locality therefore assumes that locally $\mathcal{L} = \infty$. It is necessary to investigate if it is all right to reduce all measurements to the linear approximation, especially if we leave the infinitesimal neighborhood of an event and considering that realistic measuring devices are not infinitesimal.

A restricted hypothesis of locality is the so-called *clock hypothesis*, which is a hypothesis of locality only concerned about the measurement of time. This hypothesis implies that a *standard* clock in fact measures τ, $d\tau = \sqrt{1 - \frac{v^2(t)}{c^2}}\, dt$, along its path; τ is then the *proper* time along this accelerated path. In the following, we set $\tau = 0$ when $t = 0$.

According to most experiments, the hypothesis of locality seems to be true. No experiment has yet shown the hypothesis of locality to be violated (outside of radiation effects). The main reason for this finding is that all relevant length scales in feasible experiments are very small in relation to the huge acceleration lengths of the tiny accelerations we usually experience. For instance, if we take the wavelength of light for a typical optics experiment, $\lambda \sim 10^{-7}\,\mathrm{m}$, the factor $\lambda/\mathcal{L}$ is around 10^{-23} and 10^{-20} for translational and rotational accelerations, respectively. As long as all length scales are very small compared to the acceleration lengths, it seems reasonable to assume that differences between observations by accelerated and comoving inertial observers will also be very small.

Let us now discuss the hypothesis of locality and potential problems from a conceptual viewpoint.

3.2 Problems of the Hypothesis of Locality

3.2.1 Point particles (Newton)

When we describe nature using Newton's theory, all forces and movements are determined by a second order differential equation that represents the equation of motion. This equation determines the state of a particle $(\vec{x}, \vec{v})$ once the initial condition is specified. If the position and the velocity of particles coincide, the gravitational motion will be the same. This is ingrained in the whole theory, so there is no difference between the state of an accelerated particle and its linearization, a particle with the same tangent vector, by definition. No further assumption is necessary.

Having said this, one has to keep in mind that Newton's theory only describes *point particles*. No electromagnetic radiation or intrinsically extended bodies are allowed. Classical point particles do not have any basic length scale (other than $\lambda = 0$) related with them, therefore there is no length scale to be compared with the acceleration lengths $\mathcal{L}$. Hence, the validity of the hypothesis of locality does not come as a surprise in the description of inertial effects in Newton's theory.

Similarly, geometric optics, based on the ray picture of light, has no length scale connected to it, other than $\lambda = 0$, and works fine together with the hypothesis of locality.

3.2.2 Waves

Let us consider waves now. If one inertial observer measures the frequency and the wave vector $(\omega, \vec{k})$, then we know from the Lorentz transformation that another inertial observer with a relative velocity $\vec{v}$ measures a frequency

$$\omega' = \gamma \left(\omega - \vec{v} \cdot \vec{k} \right) \tag{3.1}$$

with $\gamma := \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. If we now allow this second observer to be an arbitrarily accelerated observer and we want to assume the hypothesis of locality, then the $\vec{v}$ in the above formula becomes time-dependent, and, therefore, the frequency ω' will be time-dependent, too.

Consider now a measuring device for frequencies. It needs time to measure the frequency of the wave, since the wave has to oscillate a few times for the observer to determine its frequency or wave vector. We can measure the wave frequency with reasonable accuracy, if the velocity of the accelerating observer doesn't change much over at least a period of the wave:

$$T \left| \frac{d\vec{v}}{dt} \right| \ll v \ . \tag{3.2}$$

With $\lambda = cT$, we get $\frac{\lambda}{c} a \ll v < c$, and thus

$$\lambda \ll \frac{c^2}{a} \ , \tag{3.3}$$

where the right hand side is an acceleration length.

We only can determine the frequency of a wave under the assumption that its intrinsic scale, the wavelength, is considerably smaller than the corresponding acceleration length of the observer.

We see that an electromagnetic wave has an intrinsic scale associated with it, while a point particle has no such scale in classical theories. However, if one describes particles in quantum mechanics, they have the Compton and the de Broglie wavelengths associated with them, and similar problems arise.

The difficulties with the hypothesis of locality naturally extend to the gravitational domain as well, when the gravitoelectric and gravitomagnetic length scales need to be taken into account as described in section 2.4 of the previous chapter.

3.2.3 Charged particles

Let us forget about quantum-mechanical problems for the moment and consider a classical particle that carries a charge. As long as this particle moves on a straight line with constant velocity, it cannot radiate and there is no length related with the movement. But if this charged particle is linearly accelerated or moves on a curve, it will radiate and the wavelength of the radiation will be of order $\frac{c^2}{a}$ (with a being the linear acceleration) or $\frac{c}{\Omega}$ (with Ω the angular velocity). Therefore, our assumption of the last section is *not valid* and the Hypothesis of Locality breaks down.

The question if the charge in the special case of a uniformly accelerated charge radiates or not has led to much discussion in the literature. We will discuss this case in more detail in chapter 4.

The state of a charged particle cannot be described just by knowing position and velocity $(\vec{x}, \vec{v})$. Indeed, such a particle is described by the Abraham-Lorentz-Dirac equation

$$m\frac{d^2\vec{x}}{dt^2} - \frac{2}{3}\frac{q^2}{c^3}\frac{d^3\vec{x}}{dt^3} + \cdots = \vec{F} \ , \tag{3.4}$$

where also the third derivative of position appears.

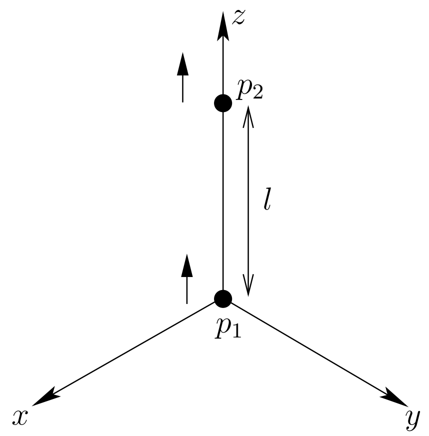

FIGURE 3.2: Two observers a distance l apart start accelerating from rest with identical acceleration profiles along the z-axis.

Another indicator for possible problems with the hypothesis of locality is the existence of intrinsic length scales for charged particles, even if they are point particles. It is natural to define the classical radius $r = \frac{q^2}{mc^2}$ of charged particles; this length appears in equation (3.4)

Quantum mechanical particles

Quantum mechanical particles have Compton and de Broglie wavelengths associated with them. The measurements of these wavelengths show the same problems as described earlier for electromagnetic waves. The general validity of the hypothesis of locality should not be assumed anymore.

3.2.4 Distances and accelerated observers

It is possible to illustrate explicitly some of the problems associated with the hypothesis of locality in the case of length measurements by accelerated observers.

Therefore, the rest of this chapter is devoted to a detailed description of certain thought experiments (Gedankenexperimente) involving observers undergoing linear (i.e. translational) and rotational accelerations.

Linear acceleration

Consider two observers that are at rest in an inertial frame and a distance l apart, see [Mas89, Mas97a]. At $t = 0$ they both start to accelerate the *same* way, according to a preplanned acceleration profile. This type of thought experiment has been considered before [Be87]. We put one of the objects at the origin of our inertial coordinate system and the other one at $(0, 0, l)$, and we assume that they accelerate linearly along the z-direction. For later calculations, we will specify the acceleration to be uniform along the z-axis, see figure 3.2. To avoid unphysical situations, we assume that the acceleration is always turned off at some finite time $t > 0$.

An inertial observer at rest in the inertial frame describes the positions of the two accelerating objects to be

$$z_{p_1}(t) = \int_0^t v(t)\, dt \;, \tag{3.5a}$$

$$z_{p_2}(t) = l + \int_0^t v(t)\, dt \;. \tag{3.5b}$$

Hence, the distance between the accelerating objects stays constant, since $z_{p_2}(t) - z_{p_1}(t) = l$.

Let us now investigate what comoving observers would measure for the distance between p_1 and p_2. The hypothesis of locality implies that both of the accelerated observers pass through the same infinite sequence of momentarily comoving inertial systems. The Lorentz transformation between the original inertial system and one of the comoving systems gives

$$l' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\, l = \gamma l \ , \tag{3.6}$$

which we generalize to

$$l' = \frac{1}{\sqrt{1-\frac{v^2(t)}{c^2}}}\, l = \gamma(t) l \ . \tag{3.7}$$

This has a simple physical interpretation: The Lorentz-Fitzgerald contracted distance between our accelerated objects is always l, hence the actual distance between them must be larger by the momentary γ-factor. It is important to recognize that p_1 and p_2 could be any two points in a measuring device that is accelerated.

Specifically, let us imagine a set of accelerated observers populating the distance between p_1 and p_2 undergoing exactly the same motions as p_1 and p_2. At any given time $\hat{t}$, each of these observers is pointwise equivalent to a comoving inertial observer in accordance with the hypothesis of locality. The Lorentz transformation connecting the global background inertial frame with the rest frame of a comoving inertial observer at $(0, 0, \hat{z})$ is given by

$$c(t-\hat{t}) = \hat{\gamma}(ct' + \hat{\beta} z') \ , \tag{3.8a}$$

$$x = x' \ , \quad y = y' \ , \quad z - \hat{z} = \hat{\gamma}(z' + c\hat{\beta} t') \ , \tag{3.8b}$$

where $\hat{\beta}$ and $\hat{\gamma}$ refer to the common speed of the system at $\hat{t}$. The consideration of length measurements of the standard observers in their inertial frames then leads to equation (3.7), i. e. the events $p_1 : (c\hat{t}, 0, 0, \hat{z}_1)$ and $p_2 : (c\hat{t}, 0, 0, \hat{z}_2)$ in the background global frame correspond to $p_1 : (ct'_1, 0, 0, z'_1)$ and $p_2 : (ct'_2, 0, 0, z'_2)$, where $ct'_1 = \hat{\gamma}\hat{\beta}(\hat{z} - \hat{z}_1)$, $z'_1 = -\hat{\gamma}(\hat{z} - \hat{z}_1)$, $ct'_2 = \hat{\gamma}\hat{\beta}(\hat{z} - \hat{z}_2)$, and $z'_2 = -\hat{\gamma}(\hat{z} - \hat{z}_2)$; therefore, $z'_2 - z'_1 = l' = \hat{\gamma}(\hat{z}_2 - \hat{z}_1) = \hat{\gamma} l$.

For an alternative description, we should be able to replace the infinite sequence of inertial systems by one system in a continuously moving frame; for example, a coordinate system that has at its spatial origin one of the accelerating objects (p_1). To this end, it is useful to introduce at this point the simplifying assumption that the observers are subject to *uniform* acceleration g. The continuously moving coordinate system (T, X, Y, Z) is related to the original inertial frame (t, x, y, z) by

$$ct = \left(Z + \frac{c^2}{g}\right) \sinh\left(\frac{gT}{c}\right), \quad x = X, \quad y = Y, \quad z = \left(Z + \frac{c^2}{g}\right) \cosh\left(\frac{gT}{c}\right) - \frac{c^2}{g} + z_0 \,. \tag{3.9}$$

We can derive a frame field from this coordinate transformation at the spatial origin (relating the derivatives ∂_μ of the two coordinate systems at this point). We get

$$\lambda^{\mu}{}_{(0)} = \left(\cosh\left(\frac{gT}{c}\right), 0, 0, \sinh\left(\frac{gT}{c}\right)\right) \,, \tag{3.10a}$$

$$\lambda^{\mu}{}_{(1)} = (0, 1, 0, 0) \,, \tag{3.10b}$$

$$\lambda^{\mu}{}_{(2)} = (0, 0, 1, 0) \,, \tag{3.10c}$$

$$\lambda^{\mu}{}_{(3)} = \left(\sinh\left(\frac{gT}{c}\right), 0, 0, \cosh\left(\frac{gT}{c}\right)\right) \,. \tag{3.10d}$$

Let us check for consistency and calculate the tensor $\Phi_{\alpha\beta}$: The nonzero covariant derivatives of the frame field are

$$\frac{D\lambda^{\mu}{}_{(0)}}{DT} = \frac{g}{c}\lambda^{\mu}{}_{(3)} \ , \qquad \frac{D\lambda^{\mu}{}_{(3)}}{DT} = \frac{g}{c}\lambda^{\mu}{}_{(0)} \ . \tag{3.11}$$

We therefore identify $\vec{a} = (0,0,g)$ and $\vec{\Omega} = (0,0,0)$, and we get as the only invariant of $\Phi_{\alpha\beta}$, see (2.16), the value $\frac{g^2}{c^4}$. Therefore, the only proper acceleration length associated with the linear acceleration is $\mathcal{L} = \frac{c^2}{g}$, as expected. The spatial frame is in fact nonrotating, i. e. it is Fermi-Walker transported along the trajectory, so that the geodesic coordinate system constructed on this basis is a Fermi system.

We now can construct an orthonormal tetrad frame along the reference trajectory such that at each instant it would coincide with the frame field of the momentary Lorentz transformation (3.8),

$$\lambda^{\mu}{}_{(0)} = (\gamma, 0, 0, \gamma\beta) \ , \tag{3.12a}$$

$$\lambda^{\mu}{}_{(1)} = (0, 1, 0, 0) \ , \tag{3.12b}$$

$$\lambda^{\mu}{}_{(2)} = (0, 0, 1, 0) \ , \tag{3.12c}$$

$$\lambda^{\mu}{}_{(3)} = (\gamma\beta, 0, 0, \gamma) \ . \tag{3.12d}$$

Using the hypothesis of locality now, we can compare the above frame fields, and thereby identify $\gamma = \cosh\left(\frac{gT}{c}\right)$ and $\beta = \tanh\left(\frac{gT}{c}\right)$. Specifically, if we describe the motion of observer p_1 as

$$t = \frac{c}{g}\sinh\left(\frac{g\tau}{c}\right) \ , \quad x = y = 0 \ , \quad z = z_0 + \frac{c^2}{g}\left(-1 + \cosh\left(\frac{g\tau}{c}\right)\right) \ , \tag{3.13}$$

where $z_0 = 0$ and τ is the proper time along the trajectory such that $\tau = 0$ at $t = 0$, we recognize the speed of the observer to be $v = c \tanh\left(\frac{g\tau}{c}\right)$. The observer p_1 is always at the spatial origin of the Fermi system with $T = \tau$ and $z_0 = 0$.

If the positions of the two accelerating objects in the original inertial frame at a time $\bar{t}$ are given by $p_1 : (c\bar{t}, 0, 0, \bar{z})$ and $p_2 : (c\bar{t}, 0, 0, l + \bar{z})$, then the corresponding positions in the moving coordinate system are $p_1 : (cT, 0, 0, 0)$ and $p_2 : (cT_2, 0, 0, L)$. From equation (3.13) we get the relations

$$c\bar{t} = \frac{c^2}{g} \sinh\left(\frac{gT}{c}\right), \quad \bar{z} = \frac{c^2}{g}\left[\cosh\left(\frac{gT}{c}\right) - 1\right] \tag{3.14}$$

and

$$c\bar{t} = \left(L + \frac{c^2}{g}\right) \sinh\left(\frac{gT_2}{c}\right), \quad \bar{z} + l = \left(L + \frac{c^2}{g}\right) \cosh\left(\frac{gT_2}{c}\right) - \frac{c^2}{g}. \tag{3.15}$$

Using $\cosh^2\Theta - \sinh^2\Theta = 1$ in the last equation yields

$$\left(L + \frac{c^2}{g}\right)^2 = \left(l + \frac{c^2}{g} + \bar{z}\right)^2 - c^2\bar{t}^2 ; \tag{3.16}$$

then, substituting for $\bar{t}$ and $\bar{z} + \frac{c^2}{g}$ using (3.14) leads to

$$\left(L + \frac{c^2}{g}\right)^2 = l^2 + 2l\frac{c^2}{g} \cosh\left(\frac{gT}{c}\right) + \left(\frac{c^2}{g}\right)^2 , \tag{3.17}$$

and this gives after some algebra

$$L = \frac{c^2}{g}\left[\sqrt{1 + 2\varepsilon\gamma + \varepsilon^2} - 1\right] = \frac{l'}{\gamma\varepsilon}\left[\sqrt{1 + 2\varepsilon\gamma + \varepsilon^2} - 1\right] \tag{3.18}$$

with $\varepsilon = {}^{l}\!/\!{}_{\frac{c^2}{g}}$ and $\gamma = \cosh\left(\frac{gT}{c}\right)$. The parameter ε compares the length l with the acceleration length in this case. For $\varepsilon \gtrsim 1$, equation (3.18) implies that L and l' can be very different; therefore, let us assume that $\varepsilon \ll 1$. We now can compare L with l', after applying the approximation $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \mathcal{O}(x^4)$ for $|x| < 1$,

$$L = \frac{l'}{\gamma\varepsilon}\left[\varepsilon\gamma + \frac{1}{2}\varepsilon^2 - \frac{1}{2}\varepsilon^2\gamma^2 - \frac{1}{2}\varepsilon^3\gamma + \frac{1}{2}\varepsilon^3\gamma^3 + \mathcal{O}(\varepsilon^4)\right] \tag{3.19}$$

$$= l'\left(1 - \frac{1}{2}\gamma\varepsilon\left(1 - \frac{1}{\gamma^2}\right) + \frac{1}{2}\gamma^2\varepsilon^2\left(1 - \frac{1}{\gamma^2}\right) + \mathcal{O}(\varepsilon^3)\right) \tag{3.20}$$

and using the relation $1 - \frac{1}{\gamma^2} = \beta^2$:

$$\frac{L}{l'} = 1 - \frac{1}{2}\beta^2\gamma\varepsilon + \frac{1}{2}\beta^2\gamma^2\varepsilon^2 + \mathcal{O}(\varepsilon^3)\ . \tag{3.21}$$

The length L measured from p_1 in this accelerated frame differs from the length l' measured in a comoving inertial frame, if the length l is not negligibly small in comparison to the acceleration length.

We now can change positions in this accelerated frame and investigate what length is measured from position p_2. Observer p_2 also follows a hyperbolic trajectory given by equation (3.13) with $z_0 = l$. The corresponding transformation between inertial coordinates and Fermi coordinates is given by (3.9) with $z_0 = l$. If the positions of the two accelerating objects in the original inertial frame at a time $\bar{t}$ are now given as before by $p_1 : (c\bar{t}, 0, 0, \bar{z})$ and $p_2 : (c\bar{t}, 0, 0, l + \bar{z})$, then the corresponding positions in the moving Fermi coordinate system are $p_1 : (cT_1, 0, 0, -L')$ and $p_2 : (cT, 0, 0, 0)$.

From equation (3.13) we get the relations

$$c\bar{t} = \left(\frac{c^2}{g} - L'\right)\sinh\left(\frac{gT_1}{c}\right), \quad \bar{z} - l = \left(\frac{c^2}{g} - L'\right)\cosh\left(\frac{gT_1}{c}\right) - \frac{c^2}{g} \tag{3.22}$$

and just as in equation (3.14),

$$c\bar{t} = \frac{c^2}{g}\sinh\left(\frac{gT}{c}\right), \quad \bar{z} = \frac{c^2}{g}\cosh\left(\frac{gT}{c}\right) - \frac{c^2}{g}\,. \tag{3.23}$$

Using $\cosh^2\Theta - \sinh^2\Theta = 1$ in equation (3.22) yields

$$\left(\frac{c^2}{g} - L'\right)^2 = \left(\frac{c^2}{g} + \bar{z} - l\right)^2 - c^2\bar{t}^2\,, \tag{3.24}$$

which after substituting for $\bar{t}$ and $\bar{z} + \frac{c^2}{g}$ using (3.23) leads to

$$\left(\frac{c^2}{g} - L'\right)^2 = l^2 - 2l\frac{c^2}{g}\cosh\left(\frac{gT}{c}\right) + \left(\frac{c^2}{g}\right)^2\,, \tag{3.25}$$

and this gives after some algebra

$$L' = \frac{c^2}{g}\left[1 - \sqrt{1 - 2\varepsilon\gamma + \varepsilon^2}\right] = \frac{l'}{\gamma\varepsilon}\left[1 - \sqrt{1 - 2\varepsilon\gamma + \varepsilon^2}\right] \tag{3.26}$$

with $\varepsilon = {}^{l}/_{\frac{c^2}{g}}$ and $\gamma = \cosh\left(\frac{gT}{c}\right)$ as above. Again, for $\varepsilon \ll 1$ let us now compare L' with l', after applying the approximation $\sqrt{1-x} = 1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 + \mathcal{O}(x^4)$ for $|x| < 1$,

$$L' = \frac{l'}{\gamma\varepsilon}\left[\varepsilon\gamma - \frac{1}{2}\varepsilon^2 + \frac{1}{2}\varepsilon^2\gamma^2 - \frac{1}{2}\varepsilon^3\gamma + \frac{1}{2}\varepsilon^3\gamma^3 + \mathcal{O}(\varepsilon^4)\right] \tag{3.27}$$

$$= l'\left(1+\frac{1}{2}\gamma\varepsilon\left(1-\frac{1}{\gamma^2}\right)+\frac{1}{2}\gamma^2\varepsilon^2\left(1-\frac{1}{\gamma^2}\right)+\mathcal{O}(\varepsilon^3)\right) \tag{3.28}$$

and using the relation $1-\frac{1}{\gamma^2}=\beta^2$:

$$\frac{L'}{l'}=1+\frac{1}{2}\beta^2\gamma\varepsilon+\frac{1}{2}\beta^2\gamma^2\varepsilon^2+\mathcal{O}(\varepsilon^3)\;. \tag{3.29}$$

The length L' measured from p_2 in this accelerated frame differs from the length L (in fact, L' is larger than L for $0<\varepsilon<1$) and from the length l', if the length l is not negligible compared to the acceleration length.

Let us now take another approach, based on our operational definition of length using electromagnetic signals, as introduced in section 2.2 on page 14: We want to measure the length by timing light rays. The relation between the measured time and the length can then be derived from the metric (2.14) for our case:

$$ds^2=-\left(1+\frac{gX^3}{c^2}\right)^2(dX^0)^2+\delta_{ij}dX^idX^j\;.$$

For light rays along the X^3- or Z-axis, $ds^2=0$, $dX^1=0$, and $dX^2=0$, and therefore:

$$0=\left(1+\frac{gZ}{c^2}\right)^2(dX^0)^2-dZ^2\;,$$

or

$$dZ=\pm\left(1+\frac{gZ}{c^2}\right)c\,dT\;.$$

After integration we get

$$cT + \text{constant} = \pm\frac{c^2}{g}\ln\left(1 + \frac{gZ}{c^2}\right) .$$

From the viewpoint of observer p_1, i.e. in the Fermi frame in which p_1 is at rest, let us suppose that the signal is emitted at time T_1^- from $Z = 0$ such that the light travels the distance $Z : 0 \to L$ and arrives at time T_2 at p_2, since that is the position of $p_2 : (T_2, 0, 0, L)$ when the light arrives, i.e. $c\ln(1 + {}^{gL}\!/_{c^2}) = g(T_2 - T_1^-)$, and then back along $Z : L \to 0$, if we assume that the light is reflected by p_2 without delay so that it returns to p_1 at T_1^+ such that $c\ln(1 + {}^{gL}\!/_{c^2}) = g(T_1^+ - T_2)$. Let us note that $T_2 = (T_1^+ + T_1^-)/2$, which is the standard synchronization condition for distant events. With $L^* = c(T_1^+ - T_1^-)/2 = {}^{c^2}\!/_g \ln(1 + {}^{gL}\!/_{c^2})$ for the length determined by light-ray timing, we find that $L^* < L$, where L is determined by rulers in the accelerated system based on the hypothesis of locality; specifically, we get using (3.21)

$$L^* = \frac{c^2}{g}\ln\left(1 + \frac{g}{c^2}\left(l' - \frac{1}{2}\beta^2\gamma\varepsilon l' + \mathcal{O}(\varepsilon^2)\right)\right) .$$

With $\frac{c^2}{g} = \frac{l}{\varepsilon} = \frac{l'}{\gamma\varepsilon}$, this yields

$$L^* = \frac{l'}{\gamma\varepsilon}\ln\left(1 + \gamma\varepsilon - \frac{1}{2}\gamma^2\beta^2\varepsilon^2 + \mathcal{O}(\varepsilon^3)\right) ,$$

and with $\ln(1 + x) = x - \frac{1}{2}x^2 + \mathcal{O}(x^3)$ for $-1 < x \leq 1$, we finally find

$$\frac{L^*}{l'} = 1 - \frac{1}{2}\gamma\varepsilon(1 + \beta^2) + \mathcal{O}(\varepsilon^2) ,$$

yet another result for the measured length if $\varepsilon \not\approx 0$.

From the viewpoint of observer p_2, i.e. in the Fermi frame in which p_2 is at rest, the thought experiment can be repeated by sending the light signal from p_2 to p_1 and back without delay; in this case, a similar analysis holds except that we have to use L' instead of L in the expression corresponding to L^*. The calculation for this case yields using (3.29)

$$\begin{aligned} \frac{L'^*}{l'} &= 1 - \frac{1}{2}\gamma\varepsilon(1-\beta^2) + \mathcal{O}(\varepsilon^2) \\ &= 1 - \frac{1}{2}\frac{\varepsilon}{\gamma} + \mathcal{O}(\varepsilon^2) \ . \end{aligned}$$

It follows from these results that consistency can be achieved only if $\varepsilon = {}^{gl}\!/_{c^2} \ll 1$ is below the level of sensitivity of the measurements of the accelerated observers.

It is possible to generalize our approach to arbitrary accelerated systems: Imagine two observers that are initially at rest in an inertial frame and subsequently move in exactly the same way for $t > 0$. A vector analogue of equation (3.5) then implies that $\vec{x}_{p_2}(t) - \vec{x}_{p_1}(t) = \vec{x}_{p_2}(0) - \vec{x}_{p_1}(0)$, so that the Euclidean length between them remains the same as measured in the inertial frame. The determination of the distance between them as measured by the accelerated observers can be discussed as in the foregoing treatment. On the other hand, it is more interesting to consider a situation where the distance between the accelerated observers is defined along a curve rather than a straight line such as for two points fixed on the rotating Earth. Therefore, in the following section we consider rotating observers and assume that the rate of rotation is uniform for the sake of simplicity.

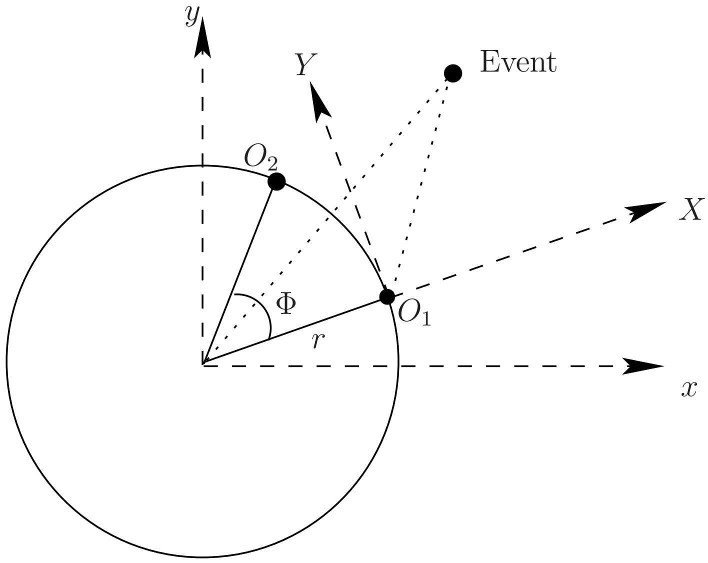

FIGURE 3.3: Two observers uniformly rotating on a circle of radius r with azimuthal angles $\phi_1 = \Omega_0 t$ and $\phi_2 = \Omega_0 t + \Phi$. An event can be described in the inertial frame (ct, x, y, z) and in a rotating geodesic coordinate system (cT, X, Y, Z).

Rotational acceleration

Similar calculations can be done for rotational motion. We consider two observers O_1 and O_2 that rotate uniformly with angular velocity Ω_0 on a circle with radius r and with a constant angle Φ between them as in figure 3.3. An inertial observer at rest in the global inertial frame would describe the arclength between the observers to have a constant length of $l = r\Phi$.

Let us now again investigate what comoving observers measure. For the sake of concreteness, we imagine a set of rotating observers populating the circle between O_1 and O_2 undergoing exactly the same motions as O_1 and O_2. The hypothesis of locality allows us to construct an infinite sequence of momentarily comoving inertial observers tangential to particles on the arc between the two circling observers. The

Lorentz transformation between the original inertial observers at rest and one of the comoving inertial observers gives infinitesimally

$$dl' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\, dl = \gamma\, dl \ , \tag{3.30}$$

with $v = r\Omega_0$. While γ in the case of uniform linear acceleration was changing, it is constant here. By integrating over the comoving inertial observers we get $l' = \gamma l$ for the arclength between the objects. The physical interpretation is the same as in the case of linear acceleration: The Lorentz-Fitzgerald contracted arclength between our rotating objects is always l, hence the actual arclength between them must be larger by the Lorentz γ-factor. Again, it is important to recognize that O_1 and O_2 could be any two points in a rotating measuring device.

Specifically, let us look at any observer between O_1 and O_2. At any given time $\hat{t}$, each of these observers is pointwise equivalent to a comoving inertial observer in accordance with the hypothesis of locality. The Lorentz transformation connecting the appropriately rotated global background inertial frame (which is at rest at each time $\hat{t}$) with the rest frame of a comoving inertial observer at $(0, 0, 0)$ is given by

$$c(t - \hat{t}) = \hat{\gamma}(ct' + \hat{\beta} y') \ , \tag{3.31a}$$

$$x = x' \ , \quad y = \hat{\gamma}(y' + c\hat{\beta} t') \ , \quad z = z' \ , \tag{3.31b}$$

where $\hat{\beta}$ and $\hat{\gamma}$ refer to the constant speed $v = r\Omega$. We now can construct an orthonormal tetrad frame along the trajectory such that at each instant it would coincide with the frame field of the momentary Lorentz transformation (3.31),

$$\lambda^{\mu}{}_{(0)} = (\gamma, 0, \gamma\beta, 0) \ , \tag{3.32a}$$

$$\lambda^{\mu}{}_{(1)} = (0, 1, 0, 0) \ , \tag{3.32b}$$

$$\lambda^{\mu}{}_{(2)} = (\gamma\beta, 0, \gamma, 0) \ , \tag{3.32c}$$

$$\lambda^{\mu}{}_{(3)} = (0, 0, 0, 1) \ . \tag{3.32d}$$

As in the case of linear acceleration, we now attempt an alternative description that is also based on the hypothesis of locality and replace the infinite sequence of momentarily comoving inertial frames by one continuously moving frame, for example, the geodesic coordinate system around the worldline of one of the rotating observers.

Let us now construct such a geodesic coordinate system for observer O_1. In equation (2.12), the worldline $\bar{x}^{\mu}(\tau)$ of O_1 is therefore given in (ct, x, y, z) coordinates by O_1 : $(ct, r\cos\varphi_1, r\sin\varphi_1, 0)$, where $t = \gamma\tau$ and $\varphi_1 = \gamma\Omega_0\tau$. Hence equation (2.12) implies that the rotating geodesic coordinate system (cT, X, Y, Z) is related to the original inertial coordinates (ct, x, y, z) by (compare figure 3.3)

$$\begin{aligned} ct = \gamma(cT + \beta Y), \quad x = (X + r)\cos(\gamma\Omega_0 T) - \gamma Y \sin(\gamma\Omega_0 T), \\ y = \gamma Y \cos(\gamma\Omega_0 T) + (X + r)\sin(\gamma\Omega_0 T), \quad Z = z \ . \end{aligned} \tag{3.33}$$

We can derive the natural orthonormal tetrad frame from this coordinate transformation (by relating the derivatives ∂_μ of the two coordinate systems). We get

$$\lambda^\mu{}_{(0)} = \gamma(1, -\beta\sin\varphi, \beta\cos\varphi, 0) \;, \tag{3.34a}$$

$$\lambda^\mu{}_{(1)} = (0, \cos\varphi, \sin\varphi, 0) \;, \tag{3.34b}$$

$$\lambda^\mu{}_{(2)} = \gamma(\beta, -\sin\varphi, \cos\varphi, 0) \;, \tag{3.34c}$$

$$\lambda^\mu{}_{(3)} = (0, 0, 0, 1) \;, \tag{3.34d}$$

where φ is the azimuthal angle of the observer such that $\frac{d\varphi}{dt} = \Omega_0$, $\beta = {}^{r\Omega_0}\!/_c$ and γ is the corresponding Lorentz factor. Let us again check for consistency and calculate the acceleration tensor $\Phi_{\alpha\beta}$: Keeping in mind that $\gamma^2(1-\beta^2) = 1$, we get for the covariant derivatives of the frame

$$\frac{D\lambda^\mu{}_{(0)}}{DT} = -\beta\gamma^2\Omega\lambda^\mu{}_{(1)} \;, \tag{3.35a}$$

$$\frac{D\lambda^\mu{}_{(1)}}{DT} = \gamma^2\Omega\lambda^\mu{}_{(2)} - \beta\gamma^2\Omega\lambda^\mu{}_{(0)} \;, \tag{3.35b}$$

$$\frac{D\lambda^\mu{}_{(2)}}{DT} = -\gamma^2\Omega\lambda^\mu{}_{(1)} \;, \tag{3.35c}$$

$$\frac{D\lambda^\mu{}_{(3)}}{DT} = 0 \;. \tag{3.35d}$$

Therefore, the non-zero components of $\Phi_{\alpha\beta}$ are $\Phi_{01} = -\Phi_{10} = -\beta\gamma^2\Omega$ and $\Phi_{12} = -\Phi_{21} = \gamma^2\Omega$, and hence the components turn out to be ${}^{\vec{a}}\!/_c = (-\beta\gamma^2\Omega_0, 0, 0)$ corresponding to the centripetal acceleration and the rotation $\vec{\Omega} = (0, 0, \gamma^2\Omega_0)$ of the spatial frame with frequency $\gamma^2\Omega_0$ about the nonrotating triad that represents ideal gyroscope directions [Mas90]. To determine the proper acceleration length in this case, we note that $I = \frac{\gamma^2\Omega_0^2}{c^2}$ and $I^* = 0$.

$$\frac{1}{c^3}\vec{a}\cdot\vec{\Omega} = 0\ , \quad \frac{-\vec{a}^2}{c^4} + \frac{\vec{\Omega}^2}{c^2} = -\frac{\beta^2\gamma^4\Omega^2}{c^2} + \frac{\gamma^4\Omega^2}{c^2} = \frac{\gamma^2\Omega^2}{c^2}\ . \tag{3.36}$$

Thus $\mathcal{L} = \frac{c}{\gamma\Omega_0}$, where $\gamma\Omega_0 = \frac{d\varphi}{d\tau}$ is the proper rotation frequency of the observer.

Consider now an observer $O : (ct, r\cos\varphi, r\sin\varphi, 0)$ on the arc between O_1 and O_2 at a given time t with $\varphi = \Omega_0 t + \phi$ such that the fixed angle ϕ could range from $\phi = 0$ at O_1 to $\phi = \Phi$ at O_2. It follows from the coordinate transformation (3.33) that in the geodesic coordinate system $O : (cT, X, Y, 0)$, where

$$X + r = r\cos\chi\ , \quad Y = \gamma^{-1} r\sin\chi\ . \tag{3.37}$$

Here χ is an angle defined by $\chi = \varphi - \gamma\Omega_0 T$; therefore, using $\varphi = \Omega_0 t + \phi$ and $t = \gamma T + \gamma\beta {}^{Y}\!/_{c}$ we find

$$\chi - \beta^2 \sin\chi = \phi\ . \tag{3.38}$$

It follows that in the geodesic coordinate system, O lies on an ellipse

$$\frac{(X+r)^2}{r^2} + \frac{Y^2}{r^2(1-\beta^2)} = 1 \tag{3.39}$$

with semimajor axis r, semiminor axis $\gamma^{-1} r$ and eccentricity $\beta = {}^{r\Omega_0}\!/_{c}$ as depicted in figure 3.4. This figure should be compared and contrasted with figure 3.3. The ellipse can be thought of as the circle of radius r that is Lorentz-Fitzgerald contracted along the direction of motion (i.e. the Y-axis). The angle χ is similar to the eccentric

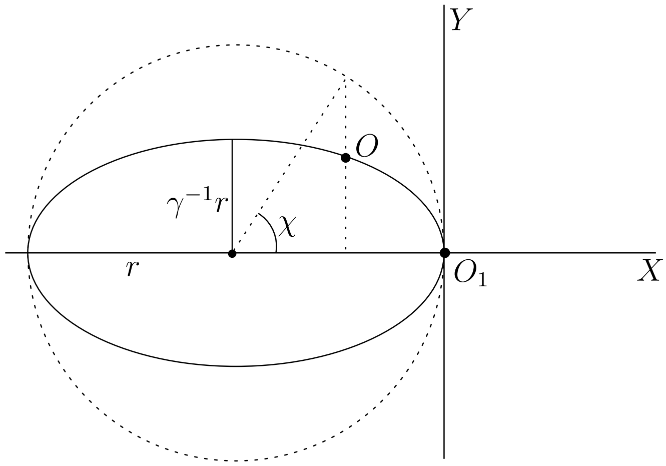

FIGURE 3.4: The observers O_1 and O are depicted here from the standpoint of the geodesic coordinate system established around the worldline of O_1. The ellipse is given by equation (3.39) and O would range from O_1 at $\chi = 0$ up to O_2 at $\chi = \Delta$, where $\Delta - \beta^2 \sin\Delta = \Phi$. The length of the elliptical arc from O_1 to O_2 is given by D in equation (3.41). This is naturally related to elliptic integrals; that is, $D = r[E(\frac{\pi}{2}, \beta) - E(\frac{\pi}{2} - \Delta, \beta)]$, where $E(\varphi, k) = \int\limits_0^{\varphi} \sqrt{1 - k^2 \sin^2\alpha}\, d\alpha$ is the elliptic integral of the second kind.

anomaly in Keplerian motion and ranges from $\chi = 0$ at O_1 to $\chi = \Delta$ at O_2, i.e.

$$\Delta - \beta^2 \sin\Delta = \Phi \tag{3.40}$$

by equation (3.38). It is interesting to note that equation (3.38) is similar to the Kepler equation for elliptical motion in Newtonian gravity, except that in the Kepler equation the eccentricity β takes the place of β^2 in (3.38). Moreover, for a given angle ϕ, there is a unique angle χ for $0 \leq \beta^2 < 1$.

In the rotating geodesic coordinate system established around O_1, the distance from O_1 to O_2 along the elliptical arc is D,

$$D = r \int_0^{\Delta} \sqrt{1 - \beta^2 \cos^2 \chi}\, d\chi \,, \tag{3.41}$$

which is in general different from $l' = \gamma r \Phi$. For instance, for a fixed Φ, $l' \to \infty$ as $\beta \to 1$, while $D \to r(1 - \cos \Delta)$ in this limit so that $D/l' \to 0$. Moreover, D is a monotonically increasing function of Φ for fixed β.

On the other hand, let us fix Φ at π and note that when $\Phi = \pi$, $\Delta = \pi$ as well from equation (3.40); then, the half circumference of the ellipse is given by

$$D = \pi r \left[1 - \left(\frac{1}{2}\right)^2 \beta^2 - \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 \frac{\beta^4}{3} - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 \frac{\beta^6}{5} - \mathcal{O}(\beta^8) \right] \,, \tag{3.42}$$

so that as β goes from $0 \to 1$, the corresponding D *decreases* from $\pi r \to 2r$ and D/l' goes from $1 \to 0$. To understand this variation intuitively, we note that $\beta\gamma = r/\mathcal{L}$. That is,

$$\frac{l'}{\mathcal{L}} = \beta \Phi \tag{3.43}$$

in the case under consideration here with $0 \le \Phi < 2\pi$. Thus, when the circular orbit is much smaller than the acceleration length of the observer, $\beta\gamma = r/\mathcal{L} \ll 1$, expanding equation (3.41) in powers of $\beta^2 \ll 1$ we find that

$$\frac{D}{l'} = 1 - \frac{3}{4}\beta^2 \left(1 + \frac{\sin 2\Phi - 8 \sin \Phi}{6\Phi}\right) + \mathcal{O}(\beta^4) \,, \tag{3.44}$$

where $\Phi = \frac{l}{r}$. When the radius of the circular orbit is much smaller than the acceleration length of the observer, $D \approx l'$; however, the deviation of $\frac{D}{l'}$ from unity cannot be neglected for $\beta \to 1$.

If the geodesic coordinate system is established along the worldline of the observer O_2 instead, then the arclength from O_2 to O_1 in the accelerated system turns out to be D as well due to the symmetry of the *uniformly* rotating configuration depicted in figure 3.3.

Considering our results, it is necessary to recognize that there is no unique answer for event distances when the observer is accelerated. We do not have a theory that gives us the precise distance on the Earth between Cologne (Germany) and Columbia (Missouri), for example, since the Earth rotates. Of course, $\varepsilon = \beta\gamma\Phi$ is typically very small, since it compares l with the very large acceleration length $\mathcal{L}$. For instance, for antipodal points along the equator, equation (3.44) implies that the difference between D and l' amounts to only a distance of the order of 10^{-3} cm.

4 Accelerated Observers and Radiation

It follows from the results of the previous chapter that in an accelerated reference frame only distances $\ll \mathcal{L}$ are well defined. In this chapter we wish to consider radiation phenomena that are based on globally defined concepts.

We first investigate the controversial question if a uniformly accelerated charge radiates. Another effect caused by acceleration is the Unruh effect, a quantum field effect that predicts that accelerated observers will see a thermal spectrum of particles.

In this chapter, we use the Minkowski metric as: $\mathrm{diag}(-1, +1, +1, +1)$.

4.1 The uniformly accelerated charge

The question if a uniformly accelerated charge radiates is a long debated and controversial question. There are several related questions, such as "Can radiation be emitted when the radiation reaction vanishes?" and "Is there a contradiction between the electromagnetic theory and the principle of equivalence in this case?"

First answers were given by Born [Bo09] in 1909 and Schott [Scho12] in 1912 independently as they calculate the electromagnetic fields of a uniformly accelerated charge in the theory of special relativity. Unfortunately, the conclusions differ: Based on Born's calculation, Pauli [WP81] and, independently, von Laue [MvL11] conclude that there is no radiation emitted, while Schott concludes that emission of radiation occurs that agrees with the standard Larmor formula for radiation.

In 1954, Bondi and Gold [BoGo55] pointed out that Born might have treated the singularity of the potentials on the light cone incorrectly. In 1960, Fulton and Rohrlich [FuRo60] discussed the problem; we agree in this dissertation with their main points. However, even in his 1997 monograph, Thirring [Th97] says: "Opinions differ as to whether a charge in hyperbolic motion emits radiation ... At this point we ... leave the reader to make up his or her own mind."

4.1.1 Potential and fields

We investigate the simple hyperbolic motion

$$t = \frac{1}{g}\sinh(g\tau) \ , \quad z = \frac{1}{g}\cosh(g\tau) \tag{4.1}$$

of a particle with electric charge e. Here we use units such that $c = 1$. We notice that $t = \pm\infty$ cannot be reached in practice. Infinite energy would be needed to accelerate the charge indefinitely; this is one of the singular aspects of this problem. We therefore employ a limiting procedure: the particle is accelerated from $-T$ to $+T$ as T becomes arbitrarily large.

The solutions for the electromagnetic potentials are the Liénard-Wiechert potentials. For the retarded potentials, they are

$$A_\mu = e\frac{v_\mu^Q}{R^\nu v_\nu^Q} \tag{4.2}$$

with Q standing for the source, and $R^\nu = (t - t_Q, \vec{R} - \vec{R}_Q)$. On the light cone $R^\nu R_\nu = 0$, and we get the *causality condition* $t - t_Q = R := |\vec{R} - \vec{R}_Q| > 0$. For the hyperbolic motion we get in cylindrical coordinates:

$$\Phi^B(\rho, z, t) = e\frac{z(\rho^2 + z^2 + \frac{1}{g^2} - t^2) - \xi t}{\xi(z^2 - t^2)} \ , \tag{4.3a}$$

$$A_\rho^B = A_\phi^B = 0 \ , \tag{4.3b}$$

$$A_z^B = e\frac{t(\rho^2 + z^2 + \frac{1}{g^2} - t^2) - \xi z}{\xi(z^2 - t^2)} \tag{4.3c}$$

with $\xi := \left[\left(\left(\frac{1}{g}\right)^2 + t^2 - \rho^2 - z^2\right)^2 + 4\left(\frac{1}{g}\right)^2 \rho^2\right]^{\frac{1}{2}}$ and the index B indicating the Born solution.

We can calculate the field strengths from these potentials. We get

$$E_\phi^B = 0 \ , \tag{4.4a}$$

$$E_z^B = -\frac{4e}{g^2}\frac{\frac{1}{g^2} + t^2 + \rho^2 - z^2}{\xi^3} \ , \tag{4.4b}$$

$$E_\rho^B = \frac{8e}{g^2}\frac{\rho z}{\xi^3} \ , \tag{4.4c}$$

$$H_\rho^B = H_z^B = 0 \ , \tag{4.4d}$$

$$H_\phi^B = \frac{8e}{g^2}\frac{\rho t}{\xi^3} \ . \tag{4.4e}$$

This is the Born solution which neglects an additional causality condition, namely, the field extends only to the region $z+t>0$. Together with the causality condition $z+t>0$, this solution was first derived by Schott [Scho12].

Bondi and Gold [BoGo55] give a solution with $F^{\mu\nu}\equiv 0$ in the half-plane $z+t<0$ and the Born solution in the half-plane $z+t>0$. At $z+t=0$ they connect both solutions in such a way that the Maxwell equations are fulfilled under that condition, too, and get:

$$E_\rho = 0\ , \quad H_\rho = H_z = 0 \tag{4.5a}$$

$$E_\rho = E_\rho^B \Theta(z+t) + \frac{2e\rho}{\rho^2+\frac{1}{g^2}}\delta(z+t) \tag{4.5b}$$

$$E_z = E_z^B \Theta(z+t) \tag{4.5c}$$

$$H_\phi = H_\phi^B \Theta(z+t) - \frac{2e\rho}{\rho^2+\frac{1}{g^2}}\delta(z+t) \tag{4.5d}$$

with

$$\Theta(x) = \begin{cases} 1, & x>0 \\ 0, & x<0 \\ \frac{1}{2}, & x=0 \end{cases} \tag{4.6}$$

The modified potentials are given by:

$$\Phi = \Phi^B \Theta(z+t) - e\ln\left(1+g^2\rho^2\right)\delta(z+t) \tag{4.7a}$$

$$A_z = A_z^B \Theta(z+t) + e\ln\left(1+g^2\rho^2\right)\delta(z+t) \tag{4.7b}$$

All solutions agree on the half-plane $z+t>0$; however they differ from each other elsewhere. The Born solution can be interpreted as the retarded solution of an accelerated charge plus the advanced solution of a mirror charge. It is plausible that cancellation effects can occur due to the additional mirror charge.

4.1.2 Radiation

To fix our interpretation, we first start with a definition of radiation. Radiation $\mathcal{R}$ (i.e. the amount of energy emitted per unit time) is the integral of the poynting vector from a fixed point of the radiation source (with time t_Q) over a surface area at a large distance:

$$\mathcal{R} := \lim_{R\to\infty} \int S \cdot \widehat{n}\, R^2\, d\Omega\ , \quad \text{fixed } t_Q\ . \tag{4.8}$$

Because we are investigating a fixed source time t_Q, the causality condition and $R \to \infty$ imply that also $t \to \infty$.

If we calculate the poynting vector,

$$S = \frac{1}{4\pi}(E \times H) = \frac{1}{4\pi} H_\phi (E_\rho \widehat{z} - E_z \widehat{\rho}) \tag{4.9}$$

using the Schott solution, we get

$$\mathcal{R} = \frac{2}{3} e^2 g^2 = \frac{2}{3} e^2 a_\mu a^\mu\ , \tag{4.10}$$

which is just the standard Larmor formula.

Pauli argued that at $t = 0$ we have $H = 0$, so the poynting vector vanishes and we have no radiation. However, because of the causality condition, the integral is taken at $R \to \infty$ and, therefore, at $t \to \infty$ as well. Radiation is a global concept, and hence can only be measured at large distances.

4.1.3 Radiation reaction

Earlier we posed the additional question “Can radiation be emitted when the radiation reaction vanishes?” For an answer we need to know the kinetic energy of the charge, and, therefore, we need to propose an equation of motion.

Abraham and Lorentz suggested that the equation of motion of a radiating charge assumed to be a sufficiently small ball of electricity with total charge e should be of the form

$$m\frac{d^2x^\mu}{dt^2} - \frac{2}{3}e^2\frac{d^3x^\mu}{dt^3} + \cdots = F^\mu_{\rm ext} \ . \tag{4.11}$$

Later, Dirac investigated the classical equation of motion of a *point* particle of charge e (like an electron), and he proposed the following equation of motion for a charged point particle:

$$ma^\mu = F^\mu_{\rm ext} + \frac{2}{3}e^2\left(\frac{d^2v^\mu}{d\tau^2} - v^\mu\frac{dv_\lambda}{d\tau}\frac{dv^\lambda}{d\tau}\right) \ , \tag{4.12}$$

where $v^\mu = \frac{dx^\mu}{d\tau}$ is the velocity vector and the term involving the parentheses is called the *radiation reaction term.* Since $v_\mu v^\mu = -1$ and $v_\mu a^\mu = 0$, this equation is consistent only if $F^\mu_{\rm ext}v_\mu = 0$, such as in the case of an external Lorentz force.

We will base our conclusions on this equation of motion. We will have to accept the consequences, or, if we don’t like the consequences, we will need to change this equation of motion.

Using the equations $a^\mu a_\mu = g^2$, $a_\mu v^\mu = 0$ and $v_\mu v^\mu = -1$, we get the relation

$\frac{a^0}{a^1} = \frac{v^1}{v^0}$, hence $-(a^1)^2 \frac{(v^1)^2}{(v^0)^2} + (a^1)^2 = g^2$. This yields

$$(a^1)^2 \left[1 - \left(\frac{v^1}{v^0}\right)^2\right] = \left(\frac{a^1}{v^0}\right)^2 = g^2; ,$$

which gives $a^1 = \pm g v^0$. Similarly, we get $a^0 = \pm g v^1$. Using these relations, we finally see that

$$\frac{d^3 x^1}{d\tau^3} = \frac{da^1}{d\tau} = \pm g \frac{dv^0}{d\tau} = \pm g a^0 = \pm g(\pm g v^1) = g^2 v^1 = g^2 \frac{dx^1}{d\tau}$$

and a similar relation for the 0-component. Therefore,

$$\frac{d^2 v^\mu}{d\tau^2} - v^\mu \underbrace{\frac{dv_\lambda}{d\tau}\frac{dv^\lambda}{d\tau}}_{=g^2} = 0 ,$$

which means that there is *no* radiation reaction in this case. This conclusion is not physically reasonable; therefore, we conclude that the Dirac form of the classical equation of motion of a radiating particle is *incomplete*.

We can write equation (4.12) for a *general* accelerated motion with $\mu = 0$ in the form

$$\frac{d}{dt}(E + Q) = \frac{1}{\gamma} F^0_{\text{ext}} - \mathcal{R} , \tag{4.13}$$

where $E = m\, {}^{dt}\!/\!_{d\tau} = m\gamma$ is the kinetic-plus-rest energy of the particle and $Q := -\frac{2}{3} e^2 a^0$ is the so-called *Schott acceleration energy*. This equation expresses the energy balance for the particle in an instantaneous form; we expect that this balance is approximately expressed by this equation due to the incomplete nature of the Dirac equation (4.12). For hyperbolic motion

$$\frac{dQ}{dt} = -\mathcal{R} \, ,$$

which implies that there is no radiation reaction.

4.1.4 Equivalence principle

We finally want to investigate the question "Is there a contradiction between the electromagnetic theory and the principle of equivalence in this case?"

Since the radiation reaction for a uniformly accelerated charge vanishes according to Dirac, both charged and uncharged particles fall equally fast in a uniform gravitational field. This appears to be in accordance with the universality of free fall. However, a freely falling particle in a gravitational field is accelerated and will radiate. On the other hand, a charged particle at rest in an accelerated elevator will *not* radiate.

There is no contradiction between the principle of equivalence and the electromagnetic theory, because the principle of equivalence is only a *locally* valid principle. However, classical electromagnetic radiation is not a local concept, that is, classical radiation cannot be measured locally. A large distance from the source is required. By definition, classical electromagnetic radiation is a *global* concept.

4.2 Unruh effect

As discussed in the previous chapter, accelerated frames cannot be defined globally. It is therefore impossible to set up quantum field theories in accelerated reference frames in the standard manner. We especially recognize that the basis for the famous Unruh effect which predicts that accelerated observers will see a thermal spectrum of particles is questionable: The effect is derived by Bogoljubov transformations between (infinitely) distant asymptotically flat regions of spacetime.

Imagine instead an observer that is accelerated in an inertial frame. As in the classical case, the measured components of quantized fields would just be projections onto the tetrad frame of the observer, e.g.

$$\hat{F}_{\alpha\beta}(\tau) = \hat{F}_{ij}\, \lambda^{i}{}_{(\alpha)}\, \lambda^{j}{}_{(\beta)}\,, \tag{4.14}$$

in accordance with the hypothesis of locality. In this approach to the problem under consideration here, we do not encounter the Unruh effect, i.e. it does not exist. To see this, we proceed by contradiction:

Suppose that a neutral accelerated observer does measure an electromagnetic field $\hat{F}_{\alpha\beta}(\tau)$ caused by the spectrum of Unruh-particles; then, equation (4.14) would imply that inertial observers must be able to measure such a field as well due to the geometric nature of the two-form $\hat{F}_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu}$. On the other hand, the simple acceleration of a totally neutral observer is not expected to produce any radiation in the inertial frame, in contradiction to our assumption. In fact, there is no experimental evidence in support of the Unruh effect at present. The contradiction probably reflects the fact that it takes an *infinite* amount of energy to keep a uniformly accelerated observer in motion for *all* time ($t : -\infty \to +\infty$), which is unphysical.

5 Electrodynamics and Accelerated Observers

In this chapter we will investigate electrodynamics as an example of a simple (Abelian) gauge theory. We will postulate different alternatives to the hypothesis of locality, discuss their physical meanings, and their consequences.

5.1 Quantum invariance condition

As discussed earlier, the Unruh effect claims that an accelerated observer sees a thermodynamical distribution of quanta. If there are quanta seen simply due to acceleration, the quantum number is clearly not conserved.

For inertial observers the quantum number is an invariant. We have a *quantum invariance condition*.

We would like to investigate what accelerated observers see when the hypothesis of locality is assumed and what they see when a nonlocal kernel term is used as an alternative hypothesis.

5.1.1 Classical thought experiment

We first assume the hypothesis of locality to be valid. Consider a uniformly rotating observer with angular velocity Ω and a perpendicularly incident circularly polarized electromagnetic wave with a frequency ω as seen by an inertial observer. Using the hypothesis of locality for the wave vector, the transformation between inertial observers results in the transverse Doppler effect

$$\omega' = \frac{\omega}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\omega$$

with $\frac{v}{c} = \frac{r\Omega}{c}$, so that only time dilation is involved. On the other hand, applying the hypothesis of locality to the electromagnetic field as in (4.14) we can conclude that the rotating observer measures the following frequencies ω^* for the electromagnetic field:

$$\omega^* = \gamma(\omega \mp \Omega) = \gamma\omega\left(1 \mp \frac{\Omega}{\omega}\right) \ , \tag{5.1}$$

where the upper sign represents a right circularly polarized (RCP) wave or a wave with positive helicity, and the lower sign represents a left circularly polarized (LCP) wave or a wave with negative helicity [Mas97a].

This formula is phenomenologically known experimentally since 1966 for microwaves and later for optical waves for small Ω [MNHS98]. It is more accurate than the Doppler effect; however, it has an unphysical consequence that by a mere rotation with $\Omega = \omega$, $\omega^* = 0$ in the RCP case, so that the wave stands completely still with respect to the rotating observer.

If we look at

$$\frac{\Omega}{\omega} = \frac{\lambda/2\pi}{c/\Omega} = \frac{\lambda/2\pi}{\mathcal{L}} ,$$

we see that the comparison between the magnitudes of the electromagnetic wavelength and the acceleration length might become important.

In (5.1) we can choose an angular velocity Ω so that ω^* is zero for the case of positive helicity. That means that we would have an electromagnetic field constant in time. The photon of the inertial frame disappears in the uniformly rotating frame.

5.1.2 Quantum invariance postulate

According to the discussion in the previous chapter, there is support for the hypothesis that the number of quanta cannot change, even when the observer is accelerated. This view is a generalization of a consequence of Lorentz invariance for inertial observers to all observers. We call this hypothesis *quantum invariance postulate*.

We conclude that the hypothesis of locality is only approximately valid and it would be useful to explore alternatives that go beyond the hypothesis of locality.

5.2 Different Nonlocal Models

5.2.1 Mashhoon model

Consider an electromagnetic radiation field F_{ij} in an inertial frame and an accelerated observer carrying an *orthonormal tetrad frame* $\lambda^i{}_{(\alpha)}(\tau)$ along its worldline. The hypothesis of locality implies that the field as measured by the observer is the projection of F_{ij} upon the frame of the instantaneously comoving inertial observer, i.e.

$$F_{\alpha\beta}(\tau) = F_{ij}\, \lambda^i{}_{(\alpha)}\, \lambda^j{}_{(\beta)}\,. \tag{5.2}$$

On the other hand, measuring the properties of the radiation field would necessitate finite intervals of time and space that would then involve the curvature of the world-line. The most general *linear* relationship between the measurements of the accelerated observer and the class of comoving inertial observers consistent with causality is

$$\mathcal{F}_{\alpha\beta}(\tau) = F_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} K_{\alpha\beta}{}^{\gamma\delta}(\tau,\tau')\, F_{\gamma\delta}(\tau')\, d\tau'\,, \tag{5.3}$$

where $\mathcal{F}_{\alpha\beta}$ is the *field actually measured*, τ_0 is the instant at which the acceleration begins and the kernel K is expected to depend on the acceleration of the observer. A nonlocal theory of accelerated observers has been developed [Mas93a, Mas93b] based on the assumptions that (i) K is a convolution-type kernel, i.e. it depends only on $\tau - \tau'$, and (ii) the radiation field never stands completely still with respect to an accelerated observer, as discussed in the previous section.

Let us repeat the postulates for (5.3) again: We assume *linearity* (because of the superposition principle in electrodynamics), and *causality*.

Form-factor nonlocality is different

In elementary particle physics a different form of nonlocality is sometimes employed. The so-called form-factor nonlocality postulates nonlocal interactions via a form factor around an event rather than local interactions at a spacetime point. These nonlocal interactions are not caused by accelerations; in fact, the particles are typically inertial.

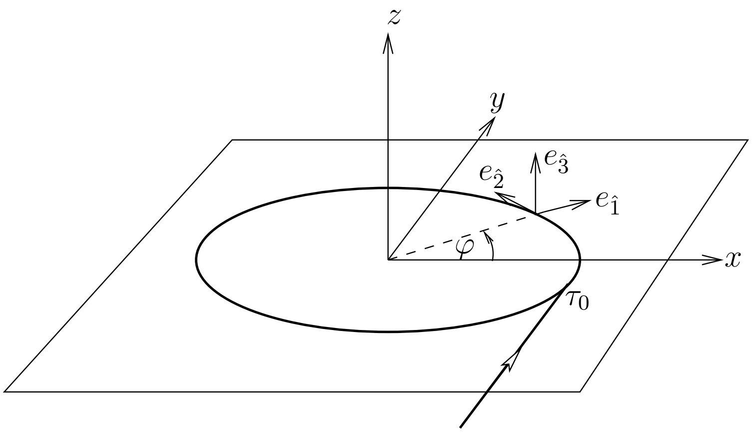

FIGURE 5.1: The path of an observer in space moving with constant angular velocity around the z-axis for $\tau > \tau_0$.

Form-factor nonlocality conflicts with our causality postulate and experiences other problems such as problems with renormalizability.

Our kernel K is not such a form factor; it can rather be described as a memory effect. While rigorous experiments regarding accumulating memory over longer periods of time are unknown to us, measuring devices are always reset to a defined state before any experiment.

Concrete example

It proves interesting to provide a concrete example of the nonlocal relationship (5.3). Imagine an observer that moves uniformly in the inertial frame along the y-axis with speed $c\beta$ for $\tau < \tau_0$ and for $\tau \geq \tau_0$ rotates with uniform angular speed Ω about the z-axis on a circle of radius r, $\beta = r\,\Omega/c$, in the (x, y)-plane, see Fig. 5.1. In this case, we have the frame (3.34) in (ct, x, y, z) coordinates with $\varphi = \Omega(t - t_0) = \gamma\,\Omega(\tau - \tau_0)$. Here φ is the azimuthal angle in the (x, y)-plane and γ is the Lorentz factor. Using six-vector notation,

$$(F_{\alpha\beta}) \to \begin{bmatrix} \hat{\boldsymbol{E}} \\ \hat{\boldsymbol{B}} \end{bmatrix}, \qquad (\mathcal{F}_{\alpha\beta}) \to \begin{bmatrix} \boldsymbol{\mathcal{E}} \\ \boldsymbol{\mathcal{B}} \end{bmatrix}, \tag{5.4}$$

one can show that with respect to the tetrad frame (3.34)

$$\boldsymbol{\mathcal{E}} = \hat{\boldsymbol{E}} + \int_{\tau_0}^{\tau} \left[\boldsymbol{\omega} \times \hat{\boldsymbol{E}}(\tau') - \frac{\boldsymbol{a}}{c} \times \hat{\boldsymbol{B}}(\tau')\right] d\tau' , \tag{5.5}$$

$$\boldsymbol{\mathcal{B}} = \hat{\boldsymbol{B}} + \int_{\tau_0}^{\tau} \left[\frac{\boldsymbol{a}}{c} \times \hat{\boldsymbol{E}}(\tau') + \boldsymbol{\omega} \times \hat{\boldsymbol{B}}(\tau')\right] d\tau' , \tag{5.6}$$

where $\boldsymbol{a}$ is the constant centripetal acceleration of the observer and $\boldsymbol{\omega}$ is its constant angular velocity. These quantities can be expressed with respect to the triad $e^i{}_A$ as $\boldsymbol{a} = (-c\beta\gamma^2\,\Omega,\, 0,\, 0)$ and $\boldsymbol{\omega} = (0,\, 0,\, \gamma^2\,\Omega)$. For an arbitrary accelerated observer, we expect that the relations analogous to (5.5) and (5.6) would be much more complicated.

In the space of continuous functions, the Volterra integral equation (5.3) provides a unique relationship between $\mathcal{F}_{\alpha\beta}$ and $F_{\alpha\beta}$. It is possible to express (5.3) as, see [CM02],

$$F_{\alpha\beta}(\tau) = \mathcal{F}_{\alpha\beta}(\tau) + \int_{\tau_0}^{\tau} R_{\alpha\beta}{}^{\gamma\delta}(\tau, \tau')\, \mathcal{F}_{\gamma\delta}(\tau')\, d\tau' , \tag{5.7}$$

where R is the resolvent kernel. If K is a convolution-type kernel as we have assumed in (i), then so is R, i.e. $R = R(\tau - \tau')$. Assumption (ii) then implies that [CM02]

$$R(\tau) = \frac{d\Lambda(\tau + \tau_0)}{d\tau}\,\Lambda^{-1}(\tau_0)\,, \tag{5.8}$$

where R and Λ are 6×6 matrices and Λ is defined by (5.2) expressed as $\hat{F} = \Lambda\, F$ in the six-vector notation. Here $\hat{F}$ denotes the field as referred to the anholonomic frame.

The kernel K can then be determined using (5.8). It turns out that this kernel is constant for the case of uniform acceleration. In particular, we obtain equations (5.5) and (5.6) for the case of a uniformly rotating observer.

5.2.2 Charge & flux model

The electrodynamics of charged particles and flux lines, see chapter 1 (compare also with [He99, Ob99] and the references cited therein), involves the electromagnetic field strength $F_{\alpha\beta}$—that is defined via the Lorentz force law and is directly related to the conservation law of magnetic flux—as well as the electromagnetic excitation $\mathcal{H}^{\alpha\beta}$ that is directly related to the electric charge conservation. The corresponding Maxwell equations are metric-free and in Ricci calculus in arbitary frames read (cf. [Scho89, Po62])

$$\partial_{[\alpha}\, F_{\beta\gamma]} - C_{[\alpha\beta}{}^{\delta}\, F_{\gamma]\delta} \;=\; 0\,, \tag{5.9}$$

$$\partial_\beta\, \mathcal{H}^{\alpha\beta} - \frac{1}{2}\, C_{\beta\gamma}{}^{\alpha}\, \mathcal{H}^{\gamma\beta} - \frac{1}{2}\, C_{\beta\gamma}{}^{\beta}\, \mathcal{H}^{\alpha\gamma} \;=\; \mathcal{J}^{\alpha}\,. \tag{5.10}$$

Here $\mathcal{J}^\alpha$ is the electric current and the C's are the components of the object of anholonomicity:

$$C_{\alpha\beta}{}^{\gamma} := 2\, e^i{}_\alpha e^j{}_\beta\, \partial_{[i}\, e_{j]}{}^{\gamma} = -C_{\beta\alpha}{}^{\gamma}\,. \tag{5.11}$$

Ordinarily for vacuum, we would have the constitutive equation

$$\mathcal{H}^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu}\, F_{\mu\nu}\,. \tag{5.12}$$

However, this reformulation of electrodynamics allows for much more general constitutive relations between $\mathcal{H}^{\alpha\beta}$ and $F_{\alpha\beta}$. In particular, it is possible to develop a nonlocal *ansatz* based on a generalization of (5.12) along the lines suggested by Obukhov and Hehl [He99]

$$\mathcal{H}^{\alpha\beta}(\tau,\xi) = \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu} \int \mathcal{K}_{\mu\nu}{}^{\rho\sigma}(\tau,\tau',\xi) F_{\rho\sigma}(\tau',\xi)\, d\tau'\;, \tag{5.13}$$

where the kernel $\mathcal{K}$ corresponds to the response of the medium and ξ^A, $A = 1, 2, 3$, are the Lagrange coordinates of the medium.

As an alternative to Mashhoon's model but along the same line of thought, see equation (5.3), one can develop an acceleration-induced nonlocal constitutive relation in vacuum via equation (5.13) by using the ansatz,

$$\begin{aligned}\mathcal{H}^{\alpha\beta}(\tau) \;=\;& \sqrt{-g}\, g^{\alpha\mu}\, g^{\beta\nu}\Big[F_{\mu\nu}(\tau)\\ &-c\int_{\tau_0}^{\tau} [\Gamma_{0\mu}{}^{\rho}(\tau-\tau')F_{\rho\nu}(\tau') + \Gamma_{0\nu}{}^{\rho}(\tau-\tau')F_{\mu\rho}(\tau')]\, d\tau'\Big]\,,\end{aligned} \tag{5.14}$$

where the integral is over the worldline of an accelerated observer in Minkowski spacetime as before. While in Mashhoon's model the measurement of the field strength is influenced by acceleration, in this model the excitation is influenced due to the constitution of the vacuum. The response of the "medium" is simply given by the Levi-Civita connection of the accelerated observer in vacuum and the local constitutive relation (5.12) is recovered for *inertial* observers.

We recall that in an *orthonormal* frame the connection is equivalent to the anholonomicity, see [Scho89]:

$$\Gamma_{\alpha\beta\gamma} := g_{\gamma\delta}\,\Gamma_{\alpha\beta}{}^{\delta} = \frac{1}{2}\left(-C_{\alpha\beta\gamma} + C_{\beta\gamma\alpha} - C_{\gamma\alpha\beta}\right) = -\Gamma_{\alpha\gamma\beta}\,. \tag{5.15}$$

If we invert (5.15), we find that $C_{\alpha\beta\gamma} = -2\Gamma_{[\alpha\beta]\gamma}$.

Let us rewrite equation (2.14) with the notation in this chapter; the metric of an arbitrary observer with local 3-acceleration $\boldsymbol{a}$ and local 3-angular velocity $\boldsymbol{\omega}$ reads:

$$\begin{aligned} ds^2 = o_{\alpha\beta}\,\vartheta^\alpha \otimes \vartheta^\beta \;&=\; \left[\left(1 + \frac{\boldsymbol{a}}{c^2}\cdot\overline{\boldsymbol{x}}\right)^2 - \left(\frac{\boldsymbol{\omega}}{c}\times\overline{\boldsymbol{x}}\right)^2\right]\left(dx^{\overline{0}}\right)^2 \\ &\quad -2\left(\frac{\boldsymbol{\omega}}{c}\times\overline{\boldsymbol{x}}\right)_{\overline{A}} dx^{\overline{0}}\,dx^{\overline{A}} - \delta_{\overline{A}\overline{B}}dx^{\overline{A}}dx^{\overline{B}}\,, \end{aligned} \tag{5.16}$$

where $(\boldsymbol{\omega}\times\overline{\boldsymbol{x}})_{\overline{A}} = \varepsilon_{\overline{A}\,\overline{B}\,\overline{C}}\,\omega^{\overline{B}}\,x^{\overline{C}}$, $\boldsymbol{a} = a^{\overline{A}}\,e_{\overline{A}}$, $a^{\overline{A}} = e_i{}^{\overline{A}}\,a^i$, and the barred coordinates are the standard normal coordinates adapted to the worldline of the accelerated observer. From this the coframe ϑ^α can be found to be

$$\begin{aligned} \vartheta^{\hat{0}} &= \left(1 + \frac{\boldsymbol{a}}{c^2}\cdot\overline{\boldsymbol{x}}\right) dx^{\overline{0}} = N dx^{\overline{0}}\,, \\ \vartheta^{A} &= dx^{\overline{A}} + \left(\frac{\boldsymbol{\omega}}{c}\times\overline{\boldsymbol{x}}\right)^{\overline{A}} dx^{\overline{0}} = dx^{\overline{A}} + N^{\overline{A}}dx^{\overline{0}}\,. \end{aligned} \tag{5.17}$$

In the (3 + 1)-decomposition of spacetime, N and $N^{\overline{A}}$ are known as *lapse function* and *shift vector*, respectively.

The *orthonormal* frame $\lambda_{(\alpha)}$ can be computed by inversion:

$$\begin{aligned}
\lambda_{(\hat{0})} &= \frac{1}{1+\frac{\boldsymbol{a}}{c^2}\cdot\overline{\boldsymbol{x}}}\left[\partial_{\overline{0}} - \left(\frac{\boldsymbol{\omega}}{c}\times\overline{\boldsymbol{x}}\right)^{\overline{B}}\partial_{\overline{B}}\right],\\
\lambda_{(A)} &= \partial_{\overline{A}}\,. \qquad (5.18)
\end{aligned}$$

The frame and the coframe are orthonormal.

Starting with the coframe, we can read off the connection coefficients (for vanishing torsion) by using Cartan's first structure equation $d\vartheta^\alpha = -\Gamma_\beta{}^\alpha\wedge\vartheta^\beta$ with $\Gamma_\beta{}^\alpha = \Gamma_{\overline{i}\beta}{}^\alpha\,dx^{\overline{i}}$. By construction, the connection projected in spacelike directions vanishes, since we have spatial Cartesian laboratory coordinates. Thus we are left with the following nonvanishing connection coefficients:

$$\begin{aligned}
\Gamma_{\overline{0}\hat{0}A} &= -\Gamma_{\overline{0}A\hat{0}} = \frac{a_A}{c^2}\,,\\
\Gamma_{\overline{0}AB} &= -\Gamma_{\overline{0}BA} = \varepsilon_{ABC}\,\frac{\omega^C}{c}\,. \qquad (5.19)
\end{aligned}$$

The first index in Γ is holonomic, whereas the second and third indices are anholonomic. If we transform the first index, by means of the frame coefficients $\lambda^{\overline{i}}{}_{(\alpha)}$, into an anholonomic one, then we find the totally anholonomic connection coefficients as follows:

$$\Gamma_{\hat{0}\hat{0}A} = -\Gamma_{\hat{0}A\hat{0}} = \frac{a_A/c^2}{1+\boldsymbol{a}\cdot\overline{\boldsymbol{x}}/c^2}\,,$$

$$\Gamma_{\hat{0}AB} \;=\; -\Gamma_{\hat{0}BA} = \frac{\varepsilon_{ABC}\,\omega^C/c}{1+\boldsymbol{a}\cdot\overline{\boldsymbol{x}}/c^2}\,. \tag{5.20}$$

In general, of course, the translational acceleration $\boldsymbol{a}$ and the angular velocity $\boldsymbol{\omega}$ are *functions of time.*

Let us return to (5.14). If we study the electric sector of the theory, we find, because of (5.19),

$$\mathcal{H}^{\hat{0}B}(\tau) = o^{\hat{0}\hat{0}}o^{BD}\left[F_{\hat{0}D}(\tau) - c\int_{\tau_0}^{\tau}\left(\Gamma_{0\hat{0}}{}^{C}F_{CD} + \Gamma_{0D}{}^{C}F_{\hat{0}C}\right)\,d\tau'\right] \tag{5.21}$$

or

$$\boldsymbol{D} = \boldsymbol{E} + \int_{\tau_0}^{\tau}\left[\boldsymbol{\omega}(\tau-\tau')\times\boldsymbol{E}(\tau') - \frac{\boldsymbol{a}(\tau-\tau')}{c}\times\boldsymbol{B}(\tau')\right]d\tau'\,. \tag{5.22}$$

Similarly, for the magnetic sector, the corresponding relations read

$$\begin{aligned}\mathcal{H}^{AB} = o^{AD}o^{BE}\Big[&F_{DE}\\ &- c\int_{\tau_0}^{\tau}\Big(\Gamma_{0D}{}^{\hat{0}}F_{\hat{0}E} + \Gamma_{0D}{}^{C}F_{CE} + \Gamma_{0E}{}^{\hat{0}}F_{D\hat{0}} + \Gamma_{0E}{}^{C}F_{DC}\Big)\,d\tau'\Big]\end{aligned} \tag{5.23}$$

or

$$\boldsymbol{H} = \boldsymbol{B} + \int_{\tau_0}^{\tau}\left[\boldsymbol{\omega}(\tau-\tau')\times\boldsymbol{B}(\tau') + \frac{\boldsymbol{a}(\tau-\tau')}{c}\times\boldsymbol{E}(\tau')\right]d\tau'\,, \tag{5.24}$$

respectively. Clearly, for *constant* $\boldsymbol{a}$ and $\boldsymbol{\omega}$ our nonlocal relations (5.22) and (5.24) are the same as (5.5) and (5.6) provided we identify $\mathcal{H}$ with $\mathcal{F}$, i.e. we postulate that the field actually measured by the accelerated observer is the excitation $\mathcal{H}$. However, this agreement does not extend to the case of *non*uniform acceleration.

5.3 Comparison of the Models

To show that the new ansatz (5.14) is different from Mashhoon's ansatz (5.3) for the case of nonuniform acceleration even when we identify $\mathcal{H}$ with $\mathcal{F}$, we proceed via contradiction. That is, let us assume that $\mathcal{F}_{\alpha\beta} = \mathcal{H}_{\alpha\beta}$ and hence from (5.22) and (5.24)

$$K(\tau) = \begin{bmatrix} K_{\boldsymbol{\omega}} & -K_{\boldsymbol{a}} \\ K_{\boldsymbol{a}} & K_{\boldsymbol{\omega}} \end{bmatrix} , \tag{5.25}$$

where $K_{\boldsymbol{\omega}} = \boldsymbol{\omega}(\tau) \cdot \boldsymbol{I}$ and $K_{\boldsymbol{a}} = \boldsymbol{a}(\tau) \cdot \boldsymbol{I}/c$. Here I_A, $(I_A)_{BC} = -\varepsilon_{ABC}$, is a 3×3 matrix that is proportional to the operator of infinitesimal rotations about the e_A-axis. We must now prove that in general $R(\tau)$ given by (5.8) cannot be the resolvent kernel corresponding to $K(\tau)$ given by (5.25).

To this end, consider an observer that is accelerated at $\tau_0 = 0$ and note that for kernels of Faltung type in equations (5.3) and (5.7) we can write

$$\overline{\mathcal{F}} = (I + \overline{K})\overline{\hat{F}} \qquad \text{and} \qquad \overline{\hat{F}} = (I + \overline{R})\overline{\mathcal{F}} , \tag{5.26}$$

respectively, where $\overline{f}(s)$ is the Laplace transform of $f(\tau)$ defined by

$$\overline{f}(s) := \int_0^\infty f(\tau) e^{-s\tau} \, d\tau \tag{5.27}$$

and I is the unit 6×6 matrix. Hence, the relation between K and R may be expressed as

$$(I + \overline{K})(I + \overline{R}) = I . \tag{5.28}$$

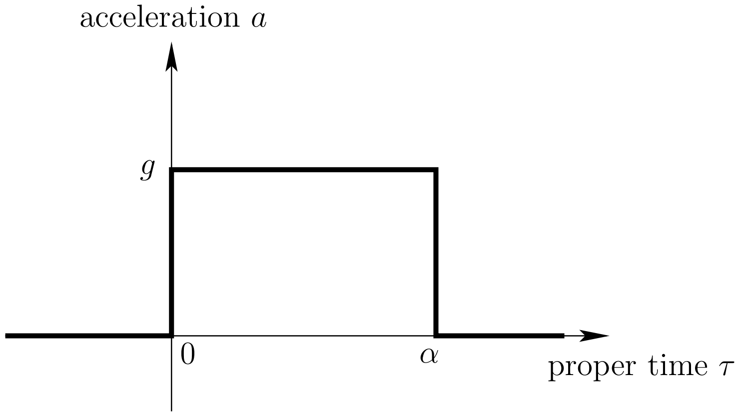

FIGURE 5.2: The acceleration of an observer that is uniformly accelerated only during a finite interval from $\tau = 0$ to $\tau = \alpha$.

Imagine now an observer that is at rest on the z-axis for $-\infty < \tau < 0$ and undergoes linear acceleration along the z-axis at $\tau = 0$ such that $a(\tau) = g > 0$ for $0 \le \tau < \alpha$ and $a(\tau) = 0$ for $\tau \ge \alpha$ (see Fig. 5.2). That is, the acceleration is turned off at $\tau = \alpha$ and thereafter the observer moves with uniform speed $c \tanh(g\alpha/c)$ along the z-axis to infinity. Thus in (5.25), $K_{\boldsymbol{\omega}} = 0$ and $K_{\boldsymbol{a}} = a(\tau)\, I_3/c$. On the other hand, one can show that (5.8) can be expressed in this case as

$$R(\tau) = a(\tau) \begin{bmatrix} U & V \\ -V & U \end{bmatrix}, \tag{5.29}$$

where $U = J_3 \sinh \Theta$, $V = I_3 \cosh \Theta$, and $(J_3)_{AB} = \delta_{AB} - \delta_{A3}\delta_{B3}$. Here we have set $c = 1$ and

$$\Theta(\tau) = \int_0^\tau a(\tau)\, d\tau = \begin{cases} g\,\tau, & 0 \le \tau < \alpha, \\ g\,\alpha, & \tau \ge \alpha. \end{cases} \tag{5.30}$$

It is now possible to work out (5.28) explicitly and conclude that for

$$X(s) := \overline{a(\tau)\sinh\Theta}\ , \quad Y(s) := \overline{a(\tau)\cosh\Theta}\ , \quad Z(s) := \overline{a(\tau)}\ , \tag{5.31}$$

we must have

$$X = YZ\ , \quad Y = Z(1+X)\ . \tag{5.32}$$

These relations imply that

$$Y(s) = \frac{Z(s)}{1-Z^2(s)}\ . \tag{5.33}$$

On the other hand, we have

$$Z(s) = \int_0^\infty a(\tau)e^{-s\tau}\, d\tau = \frac{g}{s}\left(1-e^{-\alpha s}\right) \tag{5.34}$$

and

$$\begin{aligned} Y(s) &= \frac{1}{2}\int_0^\infty a(\tau)\left(e^{\Theta}+e^{-\Theta}\right)e^{-s\tau}\, d\tau \\ &= \frac{g}{2}\left[\frac{1-e^{-(s-g)\alpha}}{s-g}+\frac{1-e^{-(s+g)\alpha}}{s+g}\right]\ . \end{aligned} \tag{5.35}$$

We consider only the region $s > g$ in which $X(s)$ and $Y(s)$ remain finite for $\alpha \to \infty$. Comparing (5.35) with

$$\frac{Z}{1-Z^2} = \frac{gs(1-e^{-\alpha s})}{s^2-g^2(1-e^{-\alpha s})^2}\ , \tag{5.36}$$

we find that, contrary to (5.33), they do not agree except in the $\alpha \to \infty$ limit (see

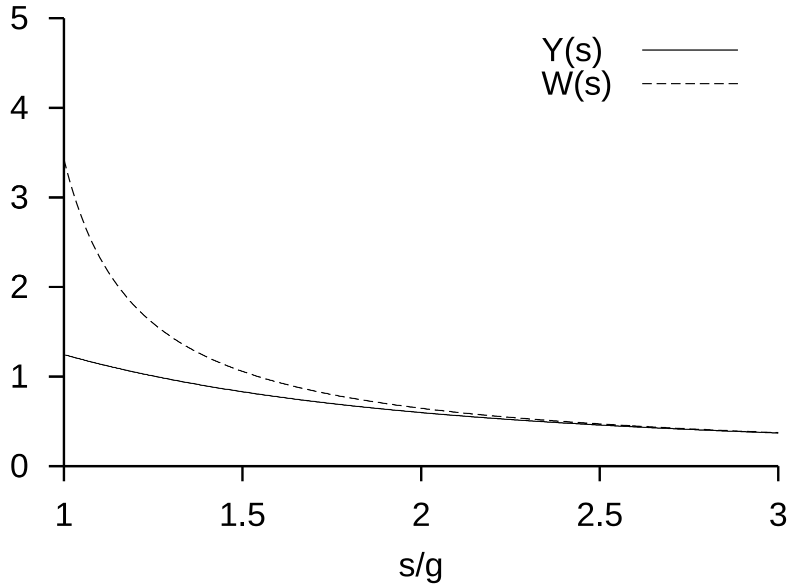

FIGURE 5.3: Plot of the functions $Y(s)$ and $W(s) := Z(s)/[1 - Z^2(s)]$ for $\alpha g = 2$.

Fig. 5.3). Therefore, we conclude that the two models are different if one considers *arbitrary* accelerations.

5.4 Nonlocality for the Electromagnetic Potential

In previous sections we suggested alternatives to the hypothesis of locality. In these alternative hypotheses nonlocal terms change the actually measured field strength or the constitutive relation. Two possibilities to get nonlocal terms were suggested: One method uses the resolvent kernel via Laplace transforms, the other method uses the connection Γ for the linear kernel K.

We now want to suggest another possibility: The nonlocal structure could enter on the level of potentials. Since the electromagnetic potential A is a connection in the electromagnetic gauge, it seems reasonable to use the gravitational connection Γ for the nonlocal term:

$$\mathcal{A}^\nu = \sqrt{-g}\, g^{\nu\mu} \left[A_\mu + c \int_{\tau_0}^{\tau} \Gamma_{0\mu}{}^{\kappa} A_\kappa \, d\tau' \right] .$$

If we write $\mathcal{A}^\mu = (\varphi, \boldsymbol{\mathcal{A}})$ and $A^\mu = (\hat{\varphi}, \hat{\boldsymbol{A}})$, then we can write

$$\varphi = \hat{\varphi} - \int_{\tau_0}^{\tau} \frac{\boldsymbol{a}(\tau - \tau')}{c} \cdot \hat{\boldsymbol{A}}(\tau')\, d\tau' \tag{5.37a}$$

$$\boldsymbol{\mathcal{A}} = \hat{\boldsymbol{A}} + \int_{\tau_0}^{\tau} \left[\boldsymbol{\omega}(\tau - \tau') \times \hat{\boldsymbol{A}} - \frac{\boldsymbol{a}(\tau - \tau')}{c}\, \hat{\varphi}(\tau') \right] d\tau' . \tag{5.37b}$$

For the case of a uniformly rotating observer, these equations coincide with the results obtained previously by Mashhoon (see the Appendix of [Mas93b]).

6 Gauge Theories of Gravity

In this chapter we want to investigate the validity of the basic assumptions of general relativity and of local gauge theories of gravity in general. A gauge theory is a theory that uses gauge potentials to preserve a global symmetry on a local level. For the most general theories of gravitation, the global symmetry is a symmetry under affine transformations. For detailed investigations of local gauge theories of gravity, such as the metric-affine theory of gravitation (MAG), see [He95]. For comments on nonlocal gauge theories of gravity (where the local gauge potentials that preserve the local symmetry depend on nonlocal field values), see chapter 8.

6.1 Basic Principles

6.1.1 Equivalence Principle

Einstein's theory of gravity is based on the *principle of equivalence*. It postulates the *local* equivalence between an observer in a gravitational field and an accelerated observer in Minkowski spacetime, see figure 6.1, i.e. both observers are postulated to measure the same physics locally.

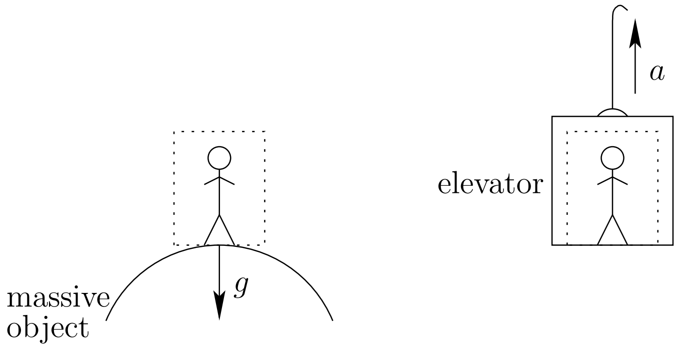

Figure 6.1: Principle of equivalence: An observer in a gravitational field and an accelerated observer in Minkowski spacetime measure the same physics *locally* (e.g. within the dotted local environment).

It is interesting to note that the question what accelerated observers measure arises at the core of GR. An observer in a gravitational field is exchanged by an accelerated observer, but we only can understand what this accelerated observer measures after we also apply the Hypothesis of Locality (or an alternative postulate that connects the measurements of accelerated and inertial observers). In other words: The measurements of an observer resting in a gravitational field get substituted by the measurements of an accelerated observer in Minkowski spacetime, i.e. after applying the hypothesis of locality for the accelerated observer at each point of its path we can compare the gravitational measurements with a series of local inertial frames and the measurements of their standard observers. If both the principle of equivalence and the hypothesis of locality are true, these two measurements will yield the same physical laws. The Minkowski spacetime of each local inertial observer can be identified with the affine tangent space at the spacetime point of the observer in the gravitational field, as we will explain in more detail in the next section.

The observer can, of course, also move in its 4-dimensional spacetime manifold.

Now the question arises: How should we connect the affine tangent spaces of the spacetime manifold? Or alternatively, using the principle of equivalence and the hypothesis of locality: How should we connect the different Minkowski spacetimes of the local observers with each other?

The simplest possibility to connect these tangent spaces is to demand that all local inertial observers live in the same global Minkowski spacetime. This can be achieved by choosing the Christoffel symbols of the manifold as connection and putting an integrability condition on these Christoffel symbols. Since the global symmetry of Minkowski spacetime is not broken locally, this procedure does not yield a gauge theory or new insights beyond accelerated observers in SR.

The most natural introduction of a gauge theory in this situation is to lift this integrability condition for the Christoffel symbols. This creates a pseudo-Riemannian spacetime with curvature and yields Einstein's theory of gravity.

But this is not the only possibility for introducing a gauge theory. One can choose more general connections than the Christoffel symbols and lift similar integrability conditions. We will investigate affine connections and corresponding gravitational gauge theories in the next sections. For example, choosing a special curvature-free connection defines a spacetime with torsion. We will deal with the corresponding teleparallel theories of gravity in chapter 7. It is possible to find a teleparallel equivalent of Einstein's theory this way.

6.1.2 Correspondence with Newtonian Gravity

Another important principle for new theories is that they have to coincide with the corresponding older (in our case Newtonian) theories in the appropriate limit. That means new theories have to agree with correct predictions of the older theories that they are trying to replace. So, in the case of gravitational theories, they have to correspond to Newton's theory in certain limits.

As for the choice of connection, the generalization of Poisson's equation

$$\nabla^2\Phi = 4\pi G\,\rho \tag{6.1}$$

can be done in several ways. The field equation of Einstein's GR seems to be the simplest generalization, but it's not the only possible solution.

6.2 Combining Affine Spaces

Let us investigate the question of how to relate neighboring affine tangent spaces in more detail, or, in other words, the question of how to connect the Minkowski spacetimes of a series of local inertial observers. The mathematical constructs we use in this section are defined in appendix A.5.

Let us start by assuming that spacetime is a sufficiently 'regular' 4-dimensional continuum, i.e. we model spacetime as a 4-dimensional connected differentiable manifold M that is Hausdorff, orientable, and paracompact. We do *not* a priori assume a metric or a connection.

Viewing the manifold M as a differentiable manifold, we can establish at any point

$p \in M$ a tangent space T_pM. According to the equivalence principle, each tangent space T_pM represents the locally equivalent Minkowski space with an observer in this tangent space who chooses an appropriately accelerated frame field. The collection of all tangent spaces T_pM yields the tangent bundle TM.

Soldering

Like every vector space, Minkowski space can be viewed as affine space. We, therefore, can enlarge any local Minkowski space T_pM to an affine tangent space A_pM by allowing to freely translate elements of T_pM to different points $p^* \in A_pM$. Similarly, the tangent bundle TM can be developed to an affine bundle AM. An affine frame of M at p is a pair (p^*, e_α). A priori, there is no point $p^* \in A_pM$ preferred, in other words, there is no origin o_p in A_pM. However, while the affine view stresses the equality of all vectors in A_pM, this can be reversed by a cut $s : U \subseteq M \to AM$ which identifies one vector in A_pM as origin. Since this origin in A_pM can be uniquely related to the point p of the manifold (using the cut s and the projection π of the tangent bundle), we call the choice of a certain cut *soldering*.

Horizontal structure

We now need a structure that relates frames in neighboring affine tangent spaces. A *horizontal structure* is such a function, defining a parallel transport between affine spaces. Specifically, we realize this parallel transport by an *affine connection* $(\Gamma^{(T)\alpha}, \Gamma^{(L)\alpha}_{\beta})$. The affine connection maps a basis (q, e_α) in A_pM onto the *affine parallel* basis of the neighboring affine tangent space $A_{\widetilde{p}}M$. With respect to the chosen basis $(\widetilde{q}, \widetilde{e}_\beta)$ in $A_{\widetilde{p}}M$, we get:

$$\text{parallel transport of } q \in A_pM = \widetilde{q} + \Gamma^{(T)\alpha}\widetilde{e}_\alpha \in A_{\widetilde{p}}M \tag{6.2a}$$

due to the translational part $\Gamma^{(T)\alpha}$ and

$$\text{parallel transport of } (e_\alpha) \in A_pM = \widetilde{e}_\alpha + \Gamma_\alpha^{(L)\beta}\widetilde{e}_\beta \in A_{\widetilde{p}}M \tag{6.2b}$$

due to the linear part $\Gamma_\beta^{(L)\alpha}$.

Exponential map and identification of neighboring points

After introducing a soldering, we can identify unique origins o_p in A_pM. Turning our focus on two neighboring affine tangent spaces, there are now two distinguished vectors in the neighboring affine tangent space $A_{\widetilde{p}}M$: the origin $o_{\widetilde{p}}$ and the horizontal parallel transport q of the origin $o_p \in A_pM$.

A third structure can be defined on affine manifolds: the *exponential map*. This is a local diffeomorphism that relates distances between points on a manifold with corresponding vectors in the affine tangential space. For neighboring affine tangent spaces we can take the differentials of a coordinate function dx^i as approximation of the exponential map. The differentials tell us the position of the origin o_p in $A_{\widetilde{p}}M$ for a vanishing translational part of the affine connection.

We now combine the horizontal structure and the exponential map:

$$\vartheta^\alpha := \Gamma^{(T)\alpha} + \delta_i^\alpha \, dx^i \tag{6.3}$$

and this yields a third point in $A_{\widetilde{p}}M$, that is not only parallel transported but also appropriately moved in order to reflect the distance between p and $\widetilde{p}$ in M. We now *identify* this point with the origin o_p in the original affine tangent space, see figure 6.2, page 82.

Torsion and Curvature

In the following we will use the tetrad ϑ^α to represent the translational part of the affine connection, and the general linear connection ${\Gamma_\alpha}^\beta$ to represent the linear part of the affine connection.

We can then define the covariant derivatives of these fields. We first look at the derivative of the tetrad which yields the *torsion* of a manifold:

$$T^\alpha := D\vartheta^\alpha = d\vartheta^\alpha + {\Gamma_\beta}^\alpha \wedge \vartheta^\beta \; . \tag{6.4}$$

The torsion describes the following property of an infinitesimally small parallelogram on a manifold: Imagine two small vectors defining a parallelogram. To complete the parallelogram we transport these original vectors parallel. The parallel transported vectors do not connect in general, the torsion describes the difference vector needed to close the parallelogram.

Similarly, we define the *curvature* of a manifold as the covariant derivative of the connection:

$$R_\alpha{}^\beta := \text{“}D{\Gamma_\alpha}^\beta\text{”} = d{\Gamma_\alpha}^\beta + {\Gamma_\gamma}^\beta \wedge {\Gamma_\alpha}^\gamma \; . \tag{6.5}$$

The curvature describes the rotation of a vector which is parallel transported along an infinitesimally small parallelogram.

Translation and Rotation

If we consider a rotational motion with acceleration length ${}^{c}\!/_{\Omega}$ in a small neighborhood, we can describe this motion in one affine tangent space. The parallel transport of the frame is mainly governed by the linear part of the affine connection, which we can identify with the regular connection of a manifold as above. In GR, this connection is in general not curvature-free.

If we consider a translation only, with acceleration length ${}^{c^2}\!/_{a}$, the frames are transported without rotating them. In this case, only the translational part of the affine connection governs the motion, and this translational part is related to the coframe ϑ^α, as described above. In general, this coframe is not torsion-free.

6.3 Lagrangian for Gauge Theories of Gravity

Let us now define a general quadratic gauge Lagrangian (this does not exclude linear terms) for gravity (without cosmological constant) that describes the metric-affine theory of gravitation (MAG):

$$\begin{aligned} V_{\mathrm{MAG}} = \frac{1}{2\ell^2} \Bigg[& -a_0\, R^{\alpha\beta} \wedge \eta_{\alpha\beta} + T^\alpha \wedge {}^\star\left(\sum_{I=1}^{3} a_I\, {}^{(I)}T_\alpha\right) \\ & + 2\left(\sum_{I=2}^{4} c_I\, {}^{(I)}Q_{\alpha\beta}\right) \wedge \vartheta^\alpha \wedge {}^\star T^\beta + Q_{\alpha\beta} \wedge {}^\star\left(\sum_{I=1}^{4} d_I\, {}^{(I)}Q^{\alpha\beta}\right)\Bigg] \\ & - \frac{1}{2} R^{\alpha\beta} \wedge {}^\star\left(\sum_{I=1}^{6} w_I\, {}^{(I)}W_{\alpha\beta} + \sum_{I=1}^{5} z_I\, {}^{(I)}Z_{\alpha\beta}\right) . \end{aligned} \tag{6.6}$$

The constants $a_0, \cdots a_3,\ c_2, \cdots, c_4,\ d_1, \cdots d_4,\ w_1, \cdots w_6$ and $z_1, \cdots z_5$ are dimensionless and ℓ is the Planck length which can be constructed from the speed of light c, the

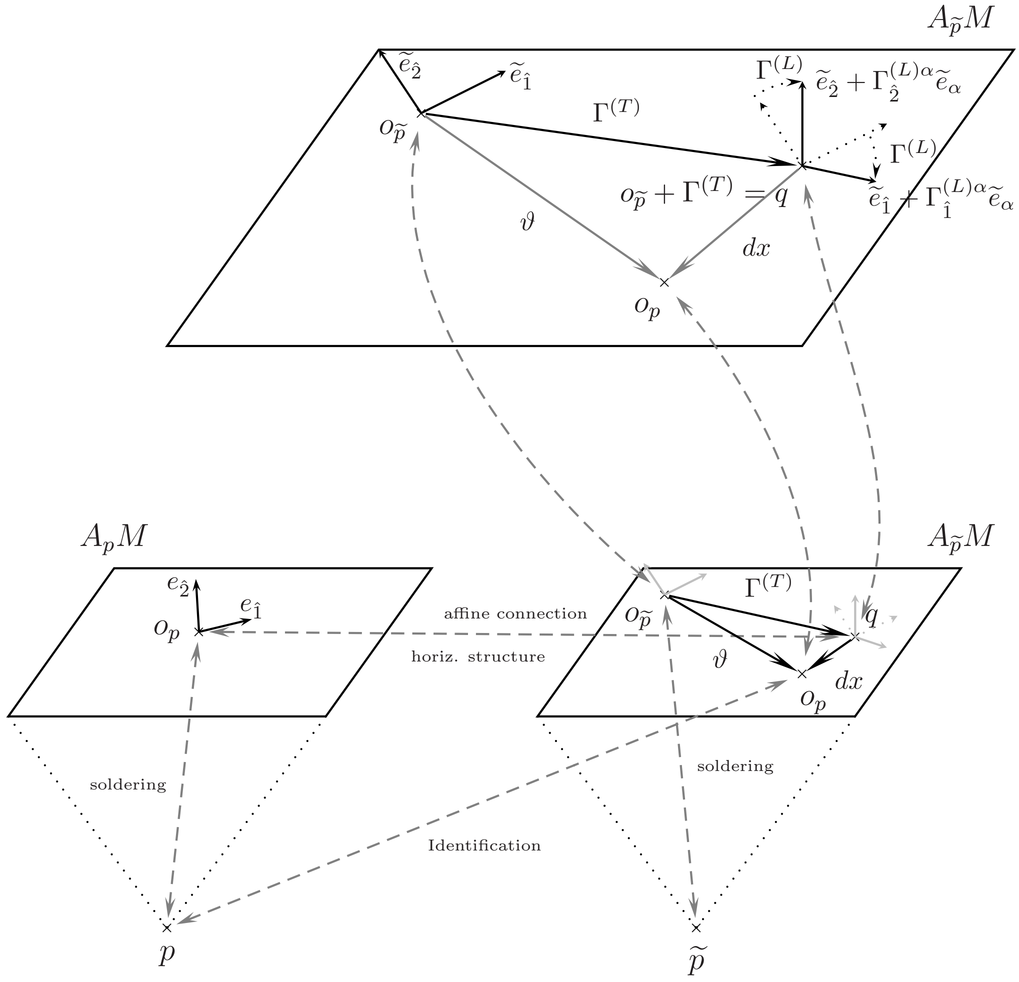

FIGURE 6.2: Two neighboring affine tangent spaces and the corresponding structures.

The soldering assigns to each point p, $\widetilde{p}$ of the manifold the origin o_p, $o_{\widetilde{p}}$ of the affine tangent spaces A_pM, $A_{\widetilde{p}}M$. The horizontal structure is implemented as affine connection $(\Gamma^{(T)\alpha}, \Gamma^{(L)\alpha}_{\beta})$. Its application on the affine basis $(o_{\widetilde{p}}, \widetilde{e}_{\alpha})$ is demonstrated in the enlarged version of $A_{\widetilde{p}}M$: The point is shifted to $q = o_{\widetilde{p}} + \Gamma^{(T)\alpha}\widetilde{e}_{\alpha}$ by the translative part $\Gamma^{(T)\alpha}$, the basis fields $\widetilde{e}_{\alpha}$ are rotated on the new basis fields $\widetilde{e}_{\beta} + \Gamma^{(L)\alpha}_{\beta}\widetilde{e}_{\alpha}$ by the linear part $\Gamma^{(L)\alpha}_{\beta}$. Finally, the exponential mapping, approximated by dx^i, leads to the cobasis ϑ^{α} via $\vartheta^{\alpha} = \Gamma^{(T)\alpha} + \delta^{\alpha}_{i}\, dx^i$. The cobasis points at the origin o_p of the neighboring affine tangent space, which can be identified with the point p of the manifold.

Einstein gravitational constant $\kappa = \frac{8\pi}{c^4}G$ and the Planck constant $\hbar$. Furthermore, $\eta_{\alpha\beta}$ is defined in (B.13), and ${}^{(I)}T_\alpha$ are the irreducible parts of the torsion, ${}^{(I)}Q_{\alpha\beta}$ are the irreducible parts of the nonmetricity, and ${}^{(I)}W_{\alpha\beta} := {}^{(I)}R_{[\alpha\beta]}$ and ${}^{(I)}Z_{\alpha\beta} := {}^{(I)}R_{(\alpha\beta)}$ are the antisymmetric and symmetric parts of the irreducible decomposition of curvature, respectively. For irreducible decompositions, see [He95, appendix B].

In this dissertation, we only want to investigate manifolds with vanishing nonmetricity. Therefore, we impose this condition by applying a Lagrange multiplier $\mu^{\alpha\beta}$, which is a symmetric 3-form, by adding the term $\frac{1}{2}Q_{\alpha\beta} \wedge \mu^{\alpha\beta}$. When varying with respect to the different gauge potentials and with respect to the Lagrange multiplier, we get the equations of motion for the theory. Due to the Lagrange multiplier term, we get the additional equation of motion $Q_{\alpha\beta} = 0$, which simplifies the other equations of motion. One recognizes that the same equations can be deduced by initially eliminating $Q_{\alpha\beta}$ in the original Lagrangian. This yields the so-called Poincaré-Lagrangian (without cosmological constant)

$$V_{\mathrm{PG}} = \frac{1}{2\ell^2}\left[-a_0 R^{\alpha\beta} \wedge \eta_{\alpha\beta} + T^\alpha \wedge {}^\star\left(\sum_{I=1}^{3} a_I\, {}^{(I)}T_\alpha\right) - \frac{1}{2}R^{\alpha\beta} \wedge {}^\star\left(\sum_{I=1}^{6} b_I\, {}^{(I)}R_{\alpha\beta}\right)\right] . \tag{6.7}$$

We can further reduce this Lagrangian by demanding that either the torsion or the curvature vanishes.

Let us first require that the torsion vanishes. We can accomplish this by adding a term $T^\alpha \wedge \lambda_\alpha$, where λ_α is a Lagrange multiplier 3-form. This yields, along a similar line of argument as for equation (6.7):

$$V_{T=0} = \frac{1}{2\ell^2}\left[-a_0 R^{\alpha\beta} \wedge \eta_{\alpha\beta} - \frac{1}{2} R^{\alpha\beta} \wedge {}^\star\left(\sum_{I=1}^{6} b_I\, {}^{(I)}R_{\alpha\beta}\right)\right] . \tag{6.8}$$

For the case $b_I = 0$ for all I, we get the Einstein-Hilbert Lagrangian, which gives Einstein's GR.

Alternatively, we can demand that the curvature should vanish. This can be accomplished by adding a term $R_\alpha{}^\beta \wedge \lambda^\alpha{}_\beta$. Here, the Lagrange multiplier $\lambda^\alpha{}_\beta$ is a 2-form with values in the space of general linear functions. We get:

$$V_{R=0} = \frac{1}{2\ell^2}\left[T^\alpha \wedge {}^\star\left(\sum_{I=1}^{3} a_I\, {}^{(I)}T_\alpha\right)\right] . \tag{6.9}$$

Gravitational theories with vanishing curvature are called teleparallel theories. We will discuss the above decomposition and other decompositions of this Lagrangian in detail in the following chapter. A specific choice of coefficients yields a teleparallel theory that is equivalent to Einstein's GR.

7 Teleparallel Theories

Gravitational theories that are defined on a curvature-free manifold with torsion are called teleparallel theories. In the previous chapter, we introduced a general teleparallel Lagrangian in equation (6.9).

Historically, different decompositions were used to split teleparallel Lagrangians into terms that could be more easily investigated. All these teleparallel Lagrangians turn out to be equivalent. In this chapter, we will first list the different decompositions with their coefficients, then we will determine the relations between the different coefficients of the different splittings, and, finally, we will investigate which coefficients lead to viable theories, i.e. to theories that are in correspondence with Newtonian gravity. We also will discuss the Lagrangians that are not in correspondence with post-Newtonian experiments, i.e. we will determine Lagrangians for which the solutions deviate from a parametrized post-Newtonian form that is standard for general relativity and is in agreement with solar system observations at the 10^{-3} level.

7.1 Lagrangians and Their Decompositions

The teleparallel Lagrangian (6.9) is quadratic in the torsion. We will now present alternative ways of splitting this quadratic term that can be used and, in fact, have been used to investigate this Lagrangian in more detail.

7.1.1 Torsion-squared in holonomic coordinates

Many investigations of gravitational theories were and are still performed in holonomic coordinates, using the notations of Ricci calculus. In this approach, the most natural splitting of the Lagrangian takes the following form:

$$V = \frac{1}{2\ell^2} \sum_{L=1}^{3} t_L \, T^{(L)} \tag{7.1}$$

with

$$T^{(1)} = T^i{}_{kl} T_i{}^{kl} \sqrt{g} \; , \tag{7.2a}$$

$$T^{(2)} = T^i{}_{ki} T_j{}^{kj} \sqrt{g} \; , \tag{7.2b}$$

$$T^{(3)} = T_{ijk} T^{jik} \sqrt{g} \; . \tag{7.2c}$$

7.1.2 Møller's tetrad theory

In 1978 Møller suggested a theory based on tetrads or basis fields. H. Meyer [Mey82] showed later that this tetrad theory is, in fact, equivalent to teleparallel theories. In order to write the teleparallel theory in Møller form, we define $\gamma_{ijk} := e_i{}^\alpha e_{k\alpha;l}$. Then the Lagrangian can be written as:

$$V = \frac{1}{8\ell^2} \sum_{k=1}^{3} \gamma_k \, L^{(k)} \tag{7.3}$$

with

$$L^{(1)} = \gamma^l{}_{il} \gamma^{mi}{}_m \sqrt{g} \, , \tag{7.4a}$$

$$L^{(2)} = \gamma_{ikl} \gamma^{ikl} \sqrt{g} \, , \tag{7.4b}$$

$$L^{(3)} = \gamma_{ikl} \gamma^{lki} \sqrt{g} \, . \tag{7.4c}$$

7.1.3 Irreducible decomposition

The teleparallel Lagrangians can be split by using the *irreducible decomposition* of the torsion (this splitting is used in (6.9) in section 6):

$$V = \frac{1}{2\ell^2} \sum_{I=1}^{3} a_I \left(D\vartheta_\alpha \wedge {}^\star {}^{(I)} D\vartheta^\alpha \right) \tag{7.5}$$

with

$$
\begin{aligned}
{}^{(1)}T^\alpha &= {}^{(1)}D\vartheta^\alpha := D\vartheta^\alpha - {}^{(2)}D\vartheta^\alpha - {}^{(3)}D\vartheta^\alpha && \text{(tentor)}, && (7.6a)\\
{}^{(2)}T^\alpha &= {}^{(2)}D\vartheta^\alpha := \frac{1}{3}\,\vartheta^\alpha \wedge \left(e_\beta \rfloor D\vartheta^\beta \right) && \text{(trator)}, && (7.6b)\\
{}^{(3)}T^\alpha &= {}^{(3)}D\vartheta^\alpha := -\frac{1}{3}\,{}^\star \left[\vartheta^\alpha \wedge {}^\star \left(\vartheta^\beta \wedge D\vartheta_\beta \right) \right] && &&\\
&= \frac{1}{3}\, e_\alpha \rfloor \left(\vartheta^\beta \wedge D\vartheta_\beta \right) && \text{(axitor)}. && (7.6c)
\end{aligned}
$$

7.1.4 Gauge-invariant Lagrangians

Finally, the teleparallel Lagrangian can be split into gauge-invariant Lagrangians which only depend on the gauge field, the coframe ϑ^α, and its first derivative, the torsion itself. We follow Rumpf [Ru78] and also include a cosmological constant term here:

$$V = \frac{1}{2\ell^2} \sum_{K=0}^{4} \rho_K \; {}^{[K]}V \; , \tag{7.7}$$

where ρ_k are constants and ρ_0 is, in fact, the cosmological constant. Then,

$$ {}^{[0]}V = \frac{1}{4}\, \vartheta^\alpha \wedge {}^\star\vartheta_\alpha = \eta \; , \tag{7.8a}$$

$$ {}^{[1]}V = D\vartheta^\alpha \wedge {}^\star D\vartheta_\alpha \qquad \text{(pure Yang-Mills type)} \; , \tag{7.8b}$$

$$ {}^{[2]}V = \left(D\vartheta_\alpha \wedge \vartheta^\alpha\right) \wedge {}^\star \left(D\vartheta_\beta \wedge \vartheta^\beta\right) \; , \tag{7.8c}$$

$$ {}^{[3]}V = \left(D\vartheta^\alpha \wedge \vartheta^\beta\right) \wedge {}^\star \left(D\vartheta_\alpha \wedge \vartheta_\beta\right) = D\vartheta^\alpha \wedge \vartheta^\beta \wedge (e_\beta \rfloor {}^\star D\vartheta_\alpha) = 2\, {}^{[1]}V \; , \tag{7.8d}$$

$$ {}^{[4]}V = \left(D\vartheta_\alpha \wedge \vartheta^\beta\right) \wedge {}^\star \left(D\vartheta_\beta \wedge \vartheta^\alpha\right) \; . \tag{7.8e}$$

7.2 Relationships between coefficients

By comparing these different splittings of the Lagrangian, we find that the different coefficients of the splittings are uniquely related to each other. Therefore, we can conclude that all the decompositions are equivalent. We will now list the relations of the coefficients of the different decompositions with the Rumpf coefficients ρ_K of the decomposition into gauge-invariant Lagrangians; this way we establish relations to transform from any splitting to any other splitting:

1. Here is the relation between the coefficients of the splitting into torsion squared terms in holonomic coordinates and the Rumpf coefficients of the splitting into gauge-invariant Lagrangians:

$$\rho_1 = 2t_1 + t_2 + t_3 \qquad t_1 = \frac{1}{2}(\rho_1 + \rho_2 + \rho_4) \tag{7.9a}$$

$$\rho_2 = -t_3 \qquad t_2 = -\rho_4 \tag{7.9b}$$

$$\rho_4 = -t_2 \qquad t_3 = -\rho_2 \; . \tag{7.9c}$$

2. The relation between the coefficients of the splitting according to Møller's tetrad theory and the Rumpf coefficients of the splitting into gauge-invariant Lagrangians follows:

$$\rho_1 = \gamma_1 + 2\gamma_2 + \gamma_3 \qquad \gamma_1 = -\rho_4 \tag{7.10a}$$

$$\rho_2 = -\frac{1}{2}\gamma_2 - \frac{1}{2}\gamma_3 \qquad \gamma_2 = \rho_1 + 2\rho_2 + \rho_4 \tag{7.10b}$$

$$\rho_4 = -\gamma_1 \qquad \gamma_3 = -\rho_1 - 4\rho_2 - \rho_4 \; . \tag{7.10c}$$

3. And finally, here is the relation between the coefficients of the splitting into irreducible parts of the torsion and the Rumpf coefficients of the splitting into gauge-invariant Lagrangians:

$$\rho_1 = \frac{1}{3}(a_2 + 2a_1) \qquad a_1 = \rho_1 + \rho_4 \tag{7.11a}$$

$$\rho_2 = \frac{1}{3}(a_3 - a_1) \qquad a_2 = \rho_1 - 2\rho_4 \tag{7.11b}$$

$$\rho_4 = \frac{1}{3}(a_1 - a_2) \qquad a_3 = \rho_1 + 3\rho_2 + \rho_4 \; . \tag{7.11c}$$

7.3 Special Choices of Coefficients

In this section we want to list and discuss several special choices for the coefficients. We will present these choices for the Rumpf coefficients in the decomposition into gauge-invariant Lagrangians.

Yang-Mills type

Investigating gravity as a gauge theory is motivated by other gauge theories whose Lagrangians are of Yang-Mills type: only the first derivative of the gauge potential appears as a squared term in the Lagrangian. Therefore, in the Yang-Mills type Lagrangian YM for teleparallel theories only the Rumpf Lagrangian of Yang-Mills type is used: $\rho_1 = 1$, $\rho_2 = \rho_4 = 0$. This yields the Lagrangian

$$V_{\mathrm{YM}} = \frac{1}{2\ell^2} T^\alpha \wedge {}^\star T_\alpha \, .$$

As we will see in the next section, the Yang-Mills type Lagrangian is not very successful for gravitational theories. Other gauge theories where the Yang-Mills approach is successful deal with exterior potentials that are defined on the manifold but are otherwise independent from the manifold. In gravity, however, the fields to be gauged are interior fields, i.e. they are properties of the manifold itself.

Other simple choices

We can take the Yang-Mills type approach and substitute the exterior derivative d by the co-derivative $d^\dagger := -{}^\star d\, {}^\star$ in the YM-Lagrangian (see section B.9 for the co-derivative). This leads to the Lagrangian $YM^{\dagger}$:

$$V_{\mathrm{YM}^{\dagger}} = \frac{1}{2\ell^2} d^{\dagger}\vartheta^{\alpha} \wedge {}^{\star}d^{\dagger}\vartheta_{\alpha}$$

with coefficients $\rho_1 = 1$, $\rho_2 = 0$, and $\rho_4 = -1$.

A similar idea was proposed by Kaniel and Itin. The Kaniel-Itin Lagrangian *KI* is the sum of the YM and YM† Lagrangians. This description represents the corrected version of the original suggestion by Kaniel and Itin [KI97], compare with [Mue98].

Viable theories

We define the *viable* teleparallel theories to be those that allow a Newtonian limit.

As we will show in the next section, viable theories demand $\rho_1 = 0$. Since multiples of a Lagrangian are equivalent to the original Lagrangian, we can choose $\rho_4 = 1$ and an arbitrary ρ_2.

The von der Heyde Lagrangian *vdH* is the most simple viable theory, as it chooses $\rho_2 = 0$, see [vdH76].

As was shown in other places (see, for example, [Mue97]), Einstein's gravitational theory is equivalent to a teleparallel version. Since Einstein's theory allows a Newtonian limit, the teleparallel version $\mathrm{GR}_{\parallel}$ is a viable theory. It turns out that the choice $\rho_2 = -\frac{1}{2}$ gives the teleparallel Lagrangian that is equivalent to the Einstein-Hilbert Lagrangian.

Summary

Let us collect the results of this section in the following table:

	$GR_{\parallel}$	vdH	viable	YM	$YM^{\dagger}$	KI
ρ_1	0	0	0	1	1	2
ρ_2	$-\frac{1}{2}$	0	arb.	0	0	0
ρ_4	1	1	1	0	−1	−1

7.4 Teleparallel Coefficients Compatible with Parametrized Post-Newtonian Theory

Theories have been developed that have a Newtonian limit and whose higher order deviations from Newton's theory are parametrized. Such theories are called *parametrized post-Newtonian theories* or PPN theories. Metrics that solve a PPN theory are called PPN solutions. In the following we will restrict ourselves to PPN solutions that describe deviations from Newtonian theory only in first order.

The parameters for such a PPN solution can be reduced to three main parameters. This leads to the metric

$$g_{00} = 1 - 2U + 2\beta U^2 \tag{7.12a}$$

$$g_{0i} = \frac{1}{2}\left(4\gamma + 4 + \alpha\right) V_i \tag{7.12b}$$

$$g_{ij} = \left(-1 - 2\gamma U\right) \delta_{ij} \tag{7.12c}$$

with

$$U = \frac{GM}{c^2 r} \quad \text{and} \quad V_i = -\frac{G}{2}\varepsilon_{ijk}\frac{x^j J^k}{c^3 r^3} \tag{7.12d}$$

for a rotating mass M with angular momentum J^k.

For Einstein's theory, the PPN parameters have the values $\gamma = 1$, $\beta = 1$, and $\alpha = 0$.

Since we are investigating teleparallel theories in this chapter, PPN metrics need to be converted to PPN coframes. We now convert the first-order metric with three parameters to a coframe via $g = g_{ij}\, dx^i \otimes dx^j = o_{\alpha\beta}\vartheta^\alpha \otimes \vartheta^\beta$. This yields

$$\vartheta^0 = a_0\, dt \;, \tag{7.13a}$$

$$\vartheta^1 = b_0\, dt + b_1\, dx \;, \tag{7.13b}$$

$$\vartheta^2 = c_0\, dt \qquad\qquad + c_2\, dy \;, \tag{7.13c}$$

$$\vartheta^3 = d_0\, dt \qquad\qquad\qquad\quad + d_3\, dz \;. \tag{7.13d}$$

with

$$a_0 \qquad = 1 - U + \left(\beta - \frac{1}{2}\right) U^2 \;, \tag{7.13e}$$

$$b_1 = c_2 = d_3 = 1 + \gamma U \;, \tag{7.13f}$$

$$b_0 = c_0 = d_0 = \frac{1}{2}\,(4\gamma + 4 + \alpha)\, V_i \;. \tag{7.13g}$$

Analysis using Computer Algebra

We used the computer algebra system Reduce to analyse the compatibility of the above ansatz for a PPN coframe with the field equations that are derived from a general teleparallel Lagrangian.

Because of limitations in computer memory and computer speed, we first analysed the spherically symmetric case and used the results to simplify the more general case of axial symmetry.

Spherically symmetric case

Considering just the spherically symmetric case, we have no rotation: $J^k = 0$, so $V_i = 0$. The parameter α is not in the metric or the coframe anymore.

Several conclusions can be drawn from the analysis with the computer algebra program REDUCE: First, it is important to note that the field equations from the second Rumpf Lagrangian (ρ_2) are always automatically fulfilled for *any* spherically symmetric ansatz. All theories with arbitrary values for ρ_2 are compatible with all spherically symmetric solutions.

More interesting conclusions can be drawn from the field equations of the other two gauge-invariant Rumpf Lagrangians. The field equations of the ρ_1-Lagrangian demand that $\gamma = 0$. On the other hand, the field equations of the ρ_4-Lagrangian demand that $\gamma = 1$.

The vanishing of γ does not allow for a Newtonian limit. Therefore, the Yang-Mills Lagrangian does *not* lead to a viable post-Newtonian theory. On the other hand, $\gamma = 1$ is, of course, compatible with Einstein's theory, which leaves arbitrary coefficients ρ_4 for possible viable teleparallel theories.

Finally, in the calculated order, β can be arbitrary and does not lead to any constraints on the Rumpf coefficients for teleparallel theories.

Axially symmetric case

In this case we allow J^k to be arbitrary. Since the ρ_1-Lagrangian has already been excluded by the spherically symmetric case, we will not consider it further.

The analysis with REDUCE gives the following results: In the calculated order, the

ρ_2-Lagrangian allows arbitrary PPN parameters. Hence, in the calculated order the ρ_2-Lagrangian is admissible for any solution.

As in the symmetrical case, the ρ_4-Lagrangian demands $\gamma = 1$. This is still compatible with Einstein's theory, leaving arbitrary coefficients ρ_4 as possibilities for viable teleparallel theories.

There are no new insights regarding any restrictions for β in the calculated order. Also, there are no restrictions for α. Both parameters can be arbitrary for any Lagrangian with $\rho_1 = 0$.

Summary

Teleparallel theories can be viewed as alternative theories for gravitation. Viable theories that are compatible with experiments can only be found when the Yang-Mills type Lagrangian vanishes. The remaining viable teleparallel theories obey $\rho_1 = 0$, $\rho_4 = 1$, and ρ_2 is arbitrary.

In the calculated order, Einstein's equivalence principle allows all viable teleparallel theories as valid alternatives to Einstein's GR. Other alternative theories, in general with non-vanishing curvature, such as metric-affine theories, could represent other valid gravitational theories. Since not every teleparallel theory is a valid gravitational theory, we expect in general restrictions for the coefficients of the general MAG-Lagrangian (6.6).

8 Summary and Conclusions

In this thesis, we have investigated some of the basic principles and foundations of gauge theories of gravity. We gave special considerations to two main assumptions, namely the hypothesis of locality and the Einstein principle of equivalence. We have also discussed concepts that are global in nature, such as radiation and alternatives to the hypothesis of locality.

The investigation of the hypothesis of locality is of importance, since it is related to very basic measurements, such as the measurement of length. The primary measurements in physics are the determinations of spatial distances and temporal durations that are associated with the effective establishment of a sufficiently local frame of reference. This process involves macrophysical determinations associated with the fact that physical observers and their frames of reference obey the laws of classical (i.e. nonquantum) physics. The basic nongravitational laws of physics refer to ideal inertial observers. On the other hand, actual observers are all (more or less) noninertial, i.e. accelerated. In fact, most experiments are performed in laboratories fixed on the Earth, which—among other motions—rotates about its axis; therefore, it is necessary to give a theoretical description of the measurements of accelerated observers. This is done via the hypothesis of locality, which in effect replaces the accelerated observer by a continuous infinity of hypothetical momentarily comoving inertial observers.

The hypothesis of locality originates from Newtonian mechanics of classical uncharged point particles. The state of such a particle is given at each instant of time by its position and velocity. It follows that the hypothesis of locality is evidently valid in Newtonian mechanics and this explains the fact that no new physical assumption is needed in Newtonian physics to deal with accelerated systems. However, as we pointed out, problems arise with the introduction of charged particles or wave phenomena. Philosophical objections to theories that lead to predetermination are also imaginable, but beyond the scope of this thesis.

It is the purpose of part of this thesis to examine critically certain basic aspects of the hypothesis of locality in connection with the measurements of accelerated observers. To this end, we studied in this work the measurement of *length* by noninertial observers. This choice is based on two considerations: (1) length measurement is a subject of crucial significance for a geometric theory of spacetime structure and (2) the hypothesis of locality must be applied not just at one event but at a continuous infinity of events for the determination of a finite length.

We have demonstrated that within the confines of classical, i.e. nonquantum physics, there exist basic limitations on length measurement by accelerated observers in Minkowski spacetime that follow from the hypothesis of locality. Indeed, realistic accelerated coordinate systems suffer from limitations that are far more severe than those imposed by the requirement of the admissibility of such coordinates. We have found that consistency can be achieved only in a rather limited neighborhood around the observer with linear dimensions that are negligibly small compared to the characteristic acceleration length of the observer.

The local acceleration scales associated with the measurements of the observer are defined via equations (2.9)–(2.11) and (2.16). These have a *physical* significance that is distinct from the acceleration radii that mark the limits of the validity of the accelerated coordinate system as can be made clear by a simple example: For observers fixed on the rotating Earth, Earth-based coordinates are essentially valid only up to the light cylinder parallel to the Earth's axis and at a radius of $c/\Omega_{\oplus} \approx 28\,\mathrm{AU}$ from it. This light cylinder, however, has no influence on the local measurements of the observer and the reception of astronomical data on the Earth. In contrast, the fact that such an observer is noninertial and therefore has local acceleration scales associated with it does affect its measurements as demonstrated by the phenomenon of spin-rotation coupling [Mas95].

Discussions of the quantum limitations of spacetime measurements are contained in [Mas89] and [SW58]. Difficulties with the measurement of spatial distance in the general theory of relativity are treated in [Schm96].

In chapter 4, we discussed effects that are globally defined, for example, we looked into the radiation of a uniformly accelerated charged particle. After investigating different opinions in the literature, we have found that a uniformly accelerated charge radiates according to the standard Larmor formula. We have also seen that global concepts such as radiation and the Unruh effect have to be viewed globally and cannot be easily discussed from a local environment.

In chapter 5, we have investigated alternatives to the hypothesis of locality, such as the Mashhoon model and the charge & flux model. We used electrodynamics as an example for a simple gauge theory.

The alternatives to the hypothesis of locality introduce nonlocal effects by taking into consideration the history of the worldline of an observer. The different alternatives we have considered give the same predictions for a uniformly rotating observer; however, they differ for arbitrary accelerations. Detailed experiments and further investigations of the principles underlying these alternatives will show if they are viable replacements for the hypothesis of locality (should future work indicate difficulties with the hypothesis of locality).

It is important to recognize that the hypothesis of locality is crucial for the physical implementation of Einstein's heuristic principle of equivalence. This cornerstone of general relativity and the hypothesis of locality *together* imply that an observer in a gravitational field is pointwise inertial.

We have shown that the principle of equivalence allows for general affine connections between neighboring affine tangential spaces. In general, this means that general metric-affine gauge theories of gravity could be viable. Specifically, we have studied teleparallel theories. One particular teleparallel theory is, in fact, equivalent to Einstein's theory of gravity. We have listed different decompositions for teleparallel Lagrangians, shown their equivalence and have investigated the viability of teleparallel Lagrangians and determined coefficients that lead to gravitational theories that are consistent with experiments up to the post-Newtonian level.

Outlook

Einstein's GR is in agreement with all *experimental* data available at present. However, future work may indicate serious difficulties with the hypothesis of locality. To go beyond this hypothesis in the theory of gravitation, one must develop a nonlocal version of Einstein's theory or of an equivalent version that would properly reduce to GR in the WKB limit for all wave phenomena. In this connection, it may be interesting to consider nonlocal gauge theories of gravitation, since Einstein's principle of equivalence is so strongly connected to the hypothesis of locality and hence would no longer be valid in the nonlocal regime.

A Definitions from differential geometry

In this appendix we display some basic definitions of differential geometry. Such definitions can be found in lecture notes [Re92] or books [KoN63] on that topic; however, slight differences might be present. This appendix, therefore, serves as collection of definitions and fixes our use of geometrical terms in this work.

A.1 Affine Space

A real n-dimensional *affine space* $A^n = (E, V, \varphi)$ consists of a set E, a real-valued n-dimensional vector space V and a map

$$\varphi : V \times E \longrightarrow E \; , \tag{A.1}$$

which obeys the following rules:

$$\varphi(0, p) = p \; , \tag{A.2a}$$

$$\varphi(v + w, p) = \varphi(v, \varphi(w, p)) \,, \tag{A.2b}$$

$$\forall p, q \in E \,, \quad \text{there is exactly one } v \in V : \; q = \varphi(v, p) \,. \tag{A.2c}$$

The set E represents a set of points, where no point is essentially different from any other point. The vector space V represents the directions in the affine space and the mapping φ tells us how one gets from one point to another point by applying a direction vector.

A.2 Manifolds

Topological Manifolds

Imagine a non-empty Hausdorff space M. We call $(U; x)$ or $(U; x_1, \ldots, x_n)$ a *map* or *coordinate neighborhood* of M, if U is a non-empty open subset of M and if

$$x = (x_1, \ldots, x_n) : U \to \mathbb{R}^n$$

is a homeomorphism on an open subset of $\mathbb{R}^n$. The individual functions $x_i : U \to \mathbb{R}$ are called *coordinate functions* of the manifold.

If there is a map $(U; x)$ for each $p \in M$ with U an open neighborhood of p, we call the set of maps an *atlas* $\mathcal{A}$. A non-empty Hausdorff space M with an atlas $\mathcal{A}$ is called a *topological manifold*.

Differentiable Manifolds

Two maps $(U;x)$ and $(V;y)$ are $\mathcal{C}^\infty$-compatible, if $U \cap V = \emptyset$ or if $U \cap V \neq \emptyset$ and the so-called *coordinate transformation* $y \circ x^{-1}$ is a $\mathcal{C}^\infty$-diffeomorphism, i.e. both $y \circ x^{-1}$ and its inversion $x \circ y^{-1}$ are infinitely differentiable. An atlas of $\mathcal{C}^\infty$-compatible maps is called a $\mathcal{C}^\infty$-atlas A^∞.

The "maximal atlas"

$$\widetilde{\mathcal{A}} := \{(U;x) \in \mathcal{A} \mid (U;x) \text{ is } \mathcal{C}^\infty\text{-compatible with all } (V;y) \in \mathcal{A}^\infty\} \tag{A.3}$$

is called the $\mathcal{C}^\infty$*-structure* induced by the atlas $\mathcal{A}$. A (paracompact) topological manifold with a $\mathcal{C}^\infty$-structure $\widetilde{\mathcal{A}}$ is called a *differentiable manifold* or $\mathcal{C}^\infty$-manifold.

A.3 Tangent Bundles

One possibility to define the tangent space makes use of differentiable curves $\alpha : J \to M$ with $J \subseteq \mathbb{R}$ being an open interval. Let us denote the set of all pairs (α, t), consisting of a curve α and a parameter $t \in J$, by $\mathcal{K}_pM$. The point p is the point of the curve α at the instant t, i.e. $\alpha(t) = p$.

Tangent Space

Let us define an equivalence relation $(\alpha, t) \sim (\beta, s)$ in $\mathcal{K}_pM$ by

$$(f \circ \alpha)'(t) = (f \circ \beta)'(s) \tag{A.4}$$

for all infinitely differentiable mappings f that are defined on a neighborhood of p.

We write $\dot{\alpha}(t)$ for the equivalence class of (α, t) and call it the *tangent vector* of the curve α at the instant t. The set of all such equivalence classes is defined to be the *tangent space* T_pM of M in p.

Tangent Space Mappings

If a mapping $f : M \to N$ between two manifolds is given, a tangent space mapping $T_pf : T_pM \to T_{f(p)}N$ can be defined by

$$T_pf\left(\dot{\alpha}(t)\right) := (f \circ \alpha)^{\cdot}(t) \ . \tag{A.5}$$

If this mapping T_pf is surjective (injective) at all points $p \in M$, then f is called a *submersion* (*immersion*).

The Tangent Bundle and the Lie Bracket

The definition of the tangent space fixes one distinct vector space for *each* point $p \in M$. This generalizes the *one* vector space V that is the same for *all* points of an affine space (see A.1). Similarly, a manifold is the generalization of the set of points of an affine space.[1] Let us now construct the union of all the vector spaces T_pM. Then, all the tangent vectors end up united in one space, the *tangent bundle*

$$TM := \bigcup_{p \in M} T_pM \ . \tag{A.6}$$

This tangent bundle TM is a manifold of dimension $2n$, if M is an n-dimensional manifold.

[1] The operation φ of an affine space is not generalized here. This is done by an affine connection, which connects neighboring (affine) tangent spaces with each other.

The *bundle projection* is a surjective mapping $\pi : TM \to M$, that assigns the point $p \in M$ to the vector $v \in T_pM$. We can introduce maps for the tangent bundle using this projection and the basis vector fields. We investigate such vector fields further in appendix B.2 on page 112.

We can define an abstract product on the tangent bundle, the so-called *Lie bracket*, by

$$\begin{aligned}[\ ,\ ]: \quad TM \times TM &\to TM \\ (u, v) &\mapsto [u, v] := uv - vu \ . \end{aligned} \tag{A.7}$$

The Lie bracket of two vector fields is again a vector field, while the two summands on their own cannot be understood as vector fields.

The Lie bracket is $\mathbb{R}$-bilinear and antisymmetric,

$$[u, v] = -[v, u] \ , \tag{A.8a}$$

and fulfills the *Jacobi identity*

$$[u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0 \ . \tag{A.8b}$$

A.4 Fiber Bundles and Vector Space Bundles

Let M and B be two $\mathcal{C}^\infty$-manifolds. A surjective submersion $\pi : M \to B$ is called a *fiber space* and $M_b := \pi^{-1}(b)$ for $b \in B$ a *fiber over* b. Furthermore, we call M *bundle*

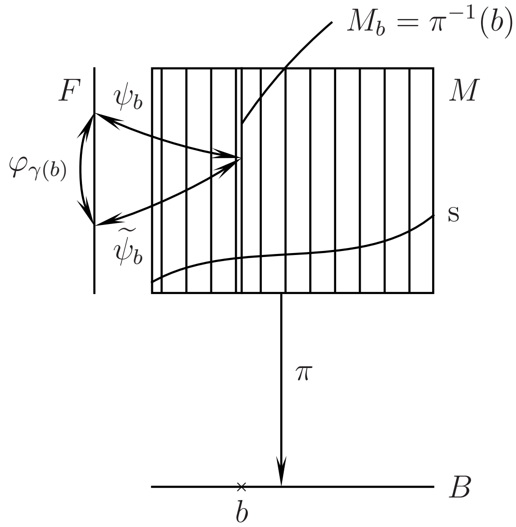

FIGURE A.1: A fiber bundle $\pi : M \to B$ with a bundle space M, base manifold B, projection π and typical fiber F. For a special fiber M_b over b two local trivializations, ψ_b and $\widetilde{\psi}_b$ and the left action $\varphi_{\gamma(b)}$ (if G-compatibility is valid) are portrayed. The curve in M is a cut $s : B \to M$.

space for which B is the base space and π is the *projection*.

Let U be an open subset of B and F be another $\mathcal{C}^\infty$-manifold. Then we call an infinitely often differentiable diffeomorphism $\psi : U \times F \to \pi^{-1}(U)$ a *local trivialization* of the fiber space $\pi : M \to B$, if $\pi \circ \psi$ is the canonical projection $\mathrm{pr}_U : U \times F \to U$. If a local trivialization exists for every *typical fiber* F and for every $b \in B$, then we call $(M, B, \pi : M \to B)$ a *fiber bundle*. A visualization of that situation is given in figure A.1.

G-Compatibilty

Let G be a *Lie group*, that is a set, which can be viewed as a manifold and as a group at the same time. In order to be consistent, the group multiplication $G \times G \to G, (g_1, g_2) \mapsto g_1 g_2 \in G$ as well as the inversion $G \to G, g \mapsto g^{-1}$ have to be continuous.

For every two local trivializations $\psi : U_1 \times F \to \pi^{-1}(U_1)$ and $\widetilde{\psi} : U_2 \times F \to \pi^{-1}(U_2)$ we have

$$\forall b \in U_1 \cap U_2 : \quad f_{\psi,\widetilde{\psi}} := \psi_b^{-1} \circ \widetilde{\psi}_b \in \mathrm{Diff}(F) \ , \tag{A.9}$$

with $\mathrm{Diff}(F)$ being called the set of diffeomorphisms on F. The two local trivializations are called *G-compatible*, if the mapping $f_{\psi,\widetilde{\psi}} : U_1 \cap U_2 \to \mathrm{Diff}(F)$ can be expressed as

$$\forall b \in U_1 \cap U_2 : \quad f_{\psi,\widetilde{\psi}} = \varphi_{\gamma(b)} \ , \tag{A.10}$$

using an infinitely often differentiable mapping $\gamma : U_1 \cap U_2 \to G$ and a left action of G on F: $\varphi : G \times F \to F$.

Vector Bundle

A fiber bundle $\pi : E \to B$ is called a *vector space bundle* or a *vector bundle*, if

- the typical fiber is a vector space V,
- every fiber E_b can be equipped with a vector space structure of the dimension of V,
- the local trivialization is linear for each fiber, i.e. ψ_b is a vector space isomorphism.

The tangent bundle TM in section A.3 is an important example of a vector bundle with $E = TM$ and $B = M$.

Cuts and Vector Fields

If a fiber bundle is given, mappings of the form $s : U \subseteq B \to E$ with $\pi \circ s = \mathrm{id}_U$ are called *cuts*, compare also with figure A.1. In case of the tangent bundle cuts

$u : M \to TM$ with $\pi \circ u = \mathrm{id}_M$ are often named *vector fields*.

A.5 Affine Tangent Bundle

Every vector space can also be viewed as an affine space (with vector addition as operation φ). If we are stressing this interpretation, we can call the tangent spaces *affine spaces* A_pM and the tangent bundle *affine tangent bundle* AM.

While the affine view stresses the equality of all vectors in A_pM, this can be reversed by a cut $s : U \subseteq M \to AM$ which identifies one vector in A_pM as origin, see figure A.1. Since this origin in A_pM can be uniquely related to the point p of the manifold (using the cut s and the projection π of the tangent bundle), we call the choice of a certain cut *soldering*.

B Definitions, Properties and Rules of Exterior Calculus

This appendix collects important rules of exterior calculus for real-valued alternating forms without proofs. Collections of such laws can also be found elsewhere (see e.g. [Hec95, He94]), proofs can be found in [Re92, Th97, ChBr82] among others. Our list serves to fix the notation, and to be an easy place for reference. Although some properties might be rarely used, we rather collected a fairly complete set of basic rules.

Notation and Prerequisites

For the rest of this appendix we will use the following symbols and assumptions unless stated otherwise:

- The $\mathcal{C}^\infty$-manifold M has *dimension* n. The space of infinitely differentiable maps over M is called $\mathcal{C}^\infty(M)$.
- We will write ψ and ϕ for alternating $\mathbb{R}$-valued forms of degree p and q, respectively, ω will be used for alternating forms of arbitrary degree. If we need

several forms of a specific degree p or q or of arbitrary degree, we number the forms: $\psi_1, \dots, \psi_i$, $\phi_1, \dots, \phi_i$, or $\omega_1, \dots, \omega_i$.

- Arbitrary vector fields are denoted by u and v, or by $u_1, \dots, u_i$ if i vector fields are needed.

B.1 Definition of Alternating Forms

A *totally antisymmetric* covariant tensor field of degree p ($p \in \mathbb{N}$) over a tangent bundle TM is called a *real-valued alternating differential form of degree* p or, for short, p-form. It is, therefore, a differentiable function

$$\psi : \overbrace{TM \times \cdots \times TM}^{p \text{ times}} \longrightarrow \mathbb{R} ,$$

which under permutation[1] π of the vector fields assumes the same sign as the sign of the permutation[2] π,

$$\psi(u_1, \dots, u_n) = \operatorname{sgn}(\pi)\, \psi\left(u_{\pi(1)}, \dots u_{\pi(n)}\right) ,$$

and which at every single point $q_0 \in M$ is a p-linear function ψ_{q_0}.

For the case $p = 0$ the above definition cannot be uniquely understood, so we add:

[1] A *permutation* π is a bijective mapping of a set $\{1, \dots, n\}$ onto itself.

[2] The *sign of a permutation* π is given by

$$\operatorname{sgn}(\pi) := \prod_{1 \le i < j \le n} \frac{\pi(j) - \pi(i)}{j - i} .$$

The $\mathcal{C}^\infty$-mappings on M are called differential forms of zero degree, or 0-forms.

This yields the *vector bundle of all p-forms over M*, i.e. according to appendix A.4 the alternating p-linear forms form a vector space at each point. We denote this vector bundle by $\Lambda^p(TM)$.

For $p > n$ the $\Lambda^p(TM)$ are bundles of null vector spaces $\{0\}$, since the vector fields that are put into ψ are not linearly independent. Because of the total antisymmetry of ψ, the alternating form has to vanish: $\psi_{p>n} \equiv 0$. The direct sum of the remaining bundles $\Lambda^p(TM)$ with $0 \leq p \leq n$ is

$$\Lambda(TM) := \bigoplus_{p=0}^{n} \Lambda^p(TM) \ .$$

B.2 Basis Fields

Maps for the tangent bundle are constructed with the projection $\pi : TM \to M$ and with the local trivialization ψ_p of isomorphisms in each fiber, which maps the T_pM on the typical fiber of vector spaces (which is simply $\mathbb{R}^n$, see appendices A.3 and A.4). For a specific construction a map $(U; x_1, \ldots, x_n)$ of M is chosen, which enables us to construct the n functions $x_i \circ \pi$, and the n coefficients of a vector in TM with respect to the n vector basis fields e_α. The vector basis fields e_α are vector fields as defined in appendix A.4, whose values $e_\alpha|_p$ at point p are linearly independent in each fiber.

The choice of vector basis fields is not distinguished from any other choice. If the vector basis fields are independent of any map $(U; x_1, \ldots, x_n)$ of the manifold, we will call e_α an *anholonomic vector basis field* or, together with $x_i \circ \pi$, *anholonomic coordinates* for TM. After a choice of a map $(U; x_1, \ldots, x_n)$ for the manifold M we can use special vector basis fields: Considering curves x_i^{-1} in the manifold, we can choose the tangent vector of the coordinate lines in p for each $p \in M$. These tangent vectors form a basis in T_pM, which is called *partial derivative fields* and symbolized by $\partial_i|_p$. This special choice of vector basis fields is called *holonomic vector basis field* or, together with $x_i \circ \pi$, *holonomic coordinates* for TM.

For the cotangent bundle $T^*M := \Lambda^1(TM)$ we choose the cobasis field ϑ^β of 1-forms that is dual to the vector basis field. This means:

$$e_\alpha \rfloor \vartheta^\beta = \vartheta^\beta(e_\alpha) = \delta^\beta_\alpha \; . \tag{B.1}$$

The coefficients of a 1-form with respect to these cobasis fields together with $x_i \circ \tau$ ($\tau : T^*M \to M$ is the projection of the cotangent bundle on M) form a map of the $2n$-dimensional cotangent bundle.

Another name for the vector basis field is *tetrad*.

Every vector field can now be decomposed with respect to the tetrad: $v = v^\alpha e_\alpha$. In particular, we can decompose the partial derivative fields of holonomic coordinates:

$$\partial_i = e_i{}^\alpha e_\alpha \; , \quad \text{with } e_i{}^\alpha \in \mathcal{C}^\infty(M) \; . \tag{B.2a}$$

In the same way we can decompose 1-forms with respect to the cobasis fields: $\psi = \psi_\beta \, \vartheta^\beta$. In particular, the differentials of coordinate functions (the dual fields to the partial derivative fields) have this decomposition:

$$dx^j = e^j{}_\beta \, \vartheta^\beta \; . \tag{B.2b}$$

On the other hand, we also can decompose arbitrary anholonomic basis and cobasis fields with respect to the special corresponding holonomic fields. We get equations similar to (B.2):

$$e_\alpha = e^i{}_\alpha \, \partial_i \qquad \text{and} \quad \vartheta^\alpha = e_i{}^\alpha \, dx^i \; . \tag{B.3}$$

Because of (B.1) we get the following conditions for the coefficient functions of the decompositions:

$$\begin{aligned} e_i{}^\gamma e^j{}_\gamma &= \delta_i^j \; , && \text{since } \delta_i^j = \partial_i \rfloor dx^j = e_i{}^\gamma \, e_\gamma \rfloor \left(e^j{}_\delta \, \vartheta^\delta \right) \; , \quad \text{and} \\ e^k{}_\beta e_k{}^\alpha &= \delta_\beta^\alpha \; , && \text{due to } \delta_\beta^\alpha = e_\beta \rfloor \vartheta^\alpha = e^k{}_\beta \, \partial_k \rfloor \left(e_l{}^\alpha \, dx^l \right) \; . \end{aligned}$$

B.3 Exterior Product $\wedge$

The exterior product is a mapping

$$\wedge : \Lambda(TM) \times \Lambda(TM) \longrightarrow \Lambda(TM)$$

with the property

$$\Lambda^p(TM) \wedge \Lambda^q(TM) \subseteq \Lambda^{p+q}(TM) \; ,$$

and is defined by

$$(\psi \wedge \phi)\,(u_1, \ldots, u_{p+q}) := \frac{1}{p!q!} \sum_{\text{perm. } \pi} \operatorname{sgn}(\pi) \psi(u_{\pi(1)}, \ldots, u_{\pi(p)}) \phi(u_{\pi(p+1)}, \ldots, u_{\pi(p+q)}) \; .$$

The exterior product is used to construct forms of higher degree from forms of lower degree.

The definition yields:

$$\text{The exterior product } \wedge \text{ is } \mathcal{C}^\infty(M)\text{-bilinear}\ , \tag{B.4a}$$

$$\text{the exterior product } \wedge \text{ is associative}\ , \tag{B.4b}$$

$$\phi \wedge \psi = (-1)^{qp}\, \psi \wedge \phi \quad \text{(graded symmetry)}\ . \tag{B.4c}$$

Using the exterior product, the so-called *monomials* $\vartheta^{\alpha_1} \wedge \cdots \wedge \vartheta^{\alpha_p}$ can be constructed from the cobasis fields. Monomials are the basis fields of the $\Lambda^p(TM)$ bundle. Hence, alternating forms can be decomposed into monomials of the cobasis:

$$\psi = \tfrac{1}{p!}\psi_{\alpha_1 \ldots \alpha_p} \vartheta^{\alpha_1} \wedge \cdots \wedge \vartheta^{\alpha_p}\ . \tag{B.5}$$

B.4 Interior Product $\rfloor$

The interior product (inserting of vector fields into forms) is denoted by $v \rfloor \omega$ or $i_v \omega$. It is a mapping

$$\rfloor : TM \times \Lambda(TM) \longrightarrow \Lambda(TM)$$

with the property

$$TM \rfloor \Lambda^p(TM) \subseteq \Lambda^{p-1}(TM)\ ,$$

and is defined by

$$(v\rfloor\psi)\,(u_1,\ldots,u_{p-1}) := \psi(v,u_1,\ldots,u_{p-1}) = p\,(\psi(v))(u_1,\ldots,u_{p-1})\;. \tag{B.6}$$

For 0-forms we demand

$$v\rfloor f \equiv 0\;.$$

These are the properties and rules of the interior product:

$$\text{The interior product } \rfloor \text{ is } \mathcal{C}^\infty(M)\text{-bilinear}\;, \tag{B.7a}$$

$$v\rfloor\,(u\rfloor\omega) = -u\rfloor\,(v\rfloor\omega)\;\;, \tag{B.7b}$$

$$v\rfloor\,(\psi\wedge\omega) = (v\rfloor\psi)\wedge\omega + (-1)^p\,\psi\wedge(v\rfloor\omega) \quad \text{(odd Leibniz rule)}\;. \tag{B.7c}$$

The rule

$$\vartheta^\mu\wedge(e_\mu\rfloor\psi) = p\,\psi \tag{B.8a}$$

is very helpful and can be derived with the expansion (B.5) and the rule (B.6). An analogous rule is

$$e_\mu\rfloor\,(\vartheta^\mu\wedge\psi) \overset{\text{(B.7c)}}{=} \delta^\mu_\mu\psi - \vartheta^\mu\wedge(e_\mu\rfloor\psi) \overset{\text{(B.8a)}}{=} (n-p)\psi\;. \tag{B.8b}$$

B.5 Metric

A symmetric non-degenerate covariant 2-tensor field over the tangent bundle TM is called a *pseudo-Riemannian metric* or, in short, a *metric*. Metrics are differentiable functions

$$g : TM \times TM \longrightarrow \mathbb{R}$$

with the properties

$$g(u,v) = g(v,u) \quad \text{(symmetry)} , \tag{B.9a}$$

$$g_{q_0} \text{ is bilinear at each point } q_0 \in M, \text{ and} \tag{B.9b}$$

$$\Big(\forall u \in T_{q_0}M : g_{q_0}(u,v) = 0\Big) \Longleftrightarrow v = 0 \quad \text{(non-degeneracy)} . \tag{B.9c}$$

As a tensor, we can decompose the metric into the tensor products of the cobasis fields:

$$g = g_{\alpha\beta}\, \vartheta^\alpha \otimes \vartheta^\beta \quad \text{with } g_{\alpha\beta} = g_{\beta\alpha} .$$

We define the number of negative eigenvalues[3] as *index* $\mathrm{Ind}(g)$ of the metric g.

B.6 Hodge Star *

The Hodge star for constructing the Hodge dual of a mapping is a function

$$^\star : \Lambda(TM) \longrightarrow \Lambda(TM)$$

with the property

$$^\star\left(\Lambda^p(TM)\right) \subseteq \Lambda^{n-p}(TM) ,$$

[3]The eigenvalues of a tensor can be found as the roots λ_i of the characteristic polynomial $\det\left(g_{\alpha\beta} - \lambda \mathbf{1}_{\alpha\beta}\right)$, with $\mathbf{1}_{\alpha\beta}$ being the unit tensor $\mathrm{diag}(\underbrace{1,\ldots,1}_{n \text{ times}})$.

defined by

$$^\star\psi := \frac{1}{(n-p)!\,p!}\sqrt{|\det g_{\mu\nu}|}\; g^{\alpha_1\gamma_1}\cdots g^{\alpha_p\gamma_p}\,\epsilon_{\alpha_1\cdots\alpha_p\beta_1\cdots\beta_{n-p}}\,\psi_{\gamma_1\cdots\gamma_p}\,\vartheta^{\beta_1}\wedge\cdots\wedge\vartheta^{\beta_{n-p}} \tag{B.10}$$

The symbol $\epsilon_{\alpha_1\cdots\alpha_n}$ represents the components of the totally antisymmetric Levi-Civita tensor density (with weight -1), i.e. $\epsilon_{0\ldots(n-1)} = +1$ in every coordinate system. The Hodge star is the only function defined here that depends on the metric $g_{\alpha\beta}$.

We now list the general rules for the Hodge star operator:

$$\text{The Hodge star } {}^\star \text{ is } \mathcal{C}^\infty(M)\text{-linear}\ , \tag{B.11a}$$

$$^{\star\star}\psi = (-1)^{p(n-p)+\mathrm{Ind}(g)}\,\psi\ , \tag{B.11b}$$

$$^\star\psi\wedge\phi = {}^\star\phi\wedge\psi \quad \text{for the } \textit{same} \text{ degree: } p = q\ . \tag{B.11c}$$

If ψ and ϕ were of different degrees, the alternating forms on the left and the right hand side would also have different degrees; therefore, equation (B.11c) would be obviously wrong.

Additionally, the following rules are valid:

$$e_\alpha\rfloor{}^\star\psi = {}^\star\left(\psi\wedge\vartheta_\alpha\right)\ . \tag{B.12a}$$

$$e_\alpha\rfloor\psi = (-1)^{n(p+1)+\mathrm{Ind}(g)}\,{}^\star\left(\vartheta_\alpha\wedge{}^\star\psi\right)\ , \tag{B.12b}$$

$$^\star\left(e_\alpha\rfloor\psi\right) = (-1)^{p-1}\vartheta_\alpha\wedge{}^\star\psi\ , \tag{B.12c}$$

$$^\star\left(e_\alpha\rfloor{}^\star\psi\right) = (-1)^{(n-1)(p+1)+\mathrm{Ind}(g)}\,\psi\wedge\vartheta_\alpha\ . \tag{B.12d}$$

Since these rules depend on each other, only one equation has to be proved. The other rules then follow easily by applying equations (B.4c) and (B.11b).

Let us define some specific Hodge-dual forms of special interest:

$$\eta := {}^\star 1 = \frac{1}{n!}\eta_{\alpha_1\ldots\alpha_n}\vartheta^{\alpha_1}\wedge\cdots\wedge\vartheta^{\alpha_n} \overset{(\mathrm{B.10})}{=} \frac{1}{n!}\sqrt{|\det g_{\mu\nu}|}\,\vartheta^{\alpha_1}\wedge\cdots\wedge\vartheta^{\alpha_n}\ , \tag{B.13a}$$

$$\begin{aligned}\eta^{\alpha_1\ldots\alpha_p} &:= {}^\star\left(\vartheta^{\alpha_1}\wedge\cdots\wedge\vartheta^{\alpha_p}\right) = \frac{1}{(n-p)!}\,\eta^{\alpha_1\ldots\alpha_p}{}_{\alpha_{p+1}\ldots\alpha_n}\vartheta^{\alpha_{p+1}}\wedge\cdots\wedge\vartheta^{\alpha_n}\\ &\overset{(\mathrm{B.10})}{=} \frac{1}{(n-p)!}\sqrt{|\det g_{\mu\nu}|}\,g^{\alpha_1\beta_1}\cdots g^{\alpha_p\beta_p}\,\epsilon_{\beta_1\cdots\beta_p\alpha_{p+1}\cdots\alpha_n}\vartheta^{\alpha_{p+1}}\wedge\cdots\wedge\vartheta^{\alpha_n}\ ,\end{aligned} \tag{B.13b}$$

$$\begin{aligned}\eta^{\alpha_1\ldots\alpha_n} &:= {}^\star\left(\vartheta^{\alpha_1}\wedge\cdots\wedge\vartheta^{\alpha_n}\right)\\ &\overset{(\mathrm{B.10})}{=} \sqrt{|\det g_{\mu\nu}|}\,g^{\alpha_1\beta_1}\cdots g^{\alpha_n\beta_n}\,\epsilon_{\beta_1\cdots\beta_n} = \frac{1}{\sqrt{|\det g_{\mu\nu}|}}\,\epsilon^{\alpha_1\cdots\alpha_n}\ .\end{aligned} \tag{B.13c}$$

The forms $\eta^{\alpha_1\cdots\alpha_p}$ form—like the monomials—basis fields of the $\Lambda^p(TM)$ spaces.

One should note that the function $\eta^{\alpha_1\cdots\alpha_n}$ only depends on the metric coefficients $g_{\mu\nu}$ and the coefficients of the Levi-Civita tensor density. An apparent additional dependency on the cobasis field might be suggested by the first line in (B.13c), but such a dependency does not exist in reality.

Because of rule (B.12a) there is the following relation between Hodge-dual forms:

$$\eta^{\alpha_1\ldots\alpha_p}{}_\mu = e_\mu\rfloor\eta^{\alpha_1\ldots\alpha_p}\ . \tag{B.14a}$$

A similar formula can be deduced from (B.12c):

$$\vartheta^\mu\wedge\eta^{\alpha_1\ldots\alpha_p} = \sum_{i=1}^{p}(-1)^{p-i}g^{\mu\alpha_i}\eta^{\alpha_1\ldots\alpha_{i-1}\alpha_{i+1}\ldots\alpha_p}\ . \tag{B.14b}$$

B.7 Exterior Derivative d

The exterior derivative is a mapping

$$d : \Lambda(TM) \longrightarrow \Lambda(TM)$$

with the property

$$d\Lambda^p(TM) \subseteq \Lambda^{p+1}(TM) \, .$$

The exterior derivative d is uniquely determined by the following rules:

$$\text{The exterior derivative } d \text{ is } \mathbb{R}\text{-linear} \, , \tag{B.15a}$$

$$d\,(\psi \wedge \omega) = (d\,\psi) \wedge \omega + (-1)^p\, \psi \wedge (d\,\omega) \quad \text{(odd Leibniz rule)} \, , \tag{B.15b}$$

$$d\,(d\,\omega) = 0 \quad \text{(nilpotency)} \, . \tag{B.15c}$$

$$\text{For 0-forms } f\text{: } df \text{ is the usual derivative.} \tag{B.15d}$$

Hence, the exterior derivative is a metric-free operation. We could have defined it earlier, but we also wanted to collect the rules for the exterior derivative of the metric and of Hodge-dual forms.

Exterior derivative of the metric determinant

For the exterior derivative of the components of the inverse metric $g^{\alpha\beta}$, which is defined by $g^{\alpha\gamma} g_{\gamma\beta} = \delta^\alpha{}_\beta$, we get the rule

$$dg^{\alpha\beta} = -g^{\alpha\gamma} g^{\delta\beta}\, dg_{\gamma\delta} \, . \tag{B.16}$$

This yields for the exterior derivative of the metric determinant:

$$d\Big(\det(g_{\mu\nu})\Big) = \det(g_{\mu\nu})\, g^{\alpha\beta}\, dg_{\alpha\beta}\ . \tag{B.17}$$

Exterior derivative of Hodge-dual forms

The formula for exterior derivatives of Hodge-dual forms is complicated to derive (see [Mue97]); it is:

$$d\eta^{\beta_1\dots\beta_p} = d\vartheta^\mu \wedge \left(e_\mu \rfloor \eta^{\beta_1\dots\beta_p}\right) + \left(\vartheta^\kappa \wedge \eta^{\beta_1\dots\beta_p\lambda} - \frac{1}{2}\, g^{\kappa\lambda}\eta^{\beta_1\dots\beta_p}\right) dg_{\kappa\lambda}\ . \tag{B.18a}$$

For the special choice of an orthonormal tetrad (i.e. $dg_{\alpha\beta} = 0$), (B.18a) yields the simpler formula

$$d\eta^{\beta_1\dots\beta_p} = d\vartheta_\mu \wedge \eta^{\beta_1\dots\beta_p\mu}\ . \tag{B.18b}$$

B.8 Lie Derivative

The *Lie derivative* of a tensor field with respect to a vector field u describes the change of a tensor field under transport along the vector field. This property can be achieved by the following definition of the Lie derivative of a vector field v along u:

$$\ell_u v = [u, v]\ . \tag{B.19a}$$

The Lie derivative for an alternating form ω is defined by the anticommuting sum of interior product and exterior derivative:

$$\ell_v \omega := v \rfloor d\omega + d(v \rfloor \omega) \,. \tag{B.19b}$$

The Lie derivative leaves the degree of a form unchanged.

The Lie derivative fulfills the following properties:

$$\text{The Lie derivative } \ell_v \text{ is } \mathbb{R}\text{-linear} \,, \tag{B.20a}$$

$$\text{For 0-forms } f\text{: } \ell_u f \text{ is the usual derivative in direction } u \,, \tag{B.20b}$$

$$\ell_v(\omega_1 \wedge \omega_2) = (\ell_v \omega_1) \wedge \omega_2 + \omega_1 \wedge (\ell_v \omega_2) \quad \text{(even Leibniz rule)} \,, \tag{B.20c}$$

$$\ell_v(d\omega) = d(\ell_v \omega) \,, \tag{B.20d}$$

$$[\ell_v, \ell_w]\, u = \ell_{[v,w]} u \quad \text{and} \qquad [\ell_v, \ell_w]\, \omega = \ell_{[v,w]} \omega \,, \tag{B.20e}$$

$$[\ell_v, i_u]\, \omega = i_{[v,u]} \omega \quad \text{or} \qquad \ell_v \,(u \rfloor \omega) - u \rfloor \ell_v \omega = [v, u] \rfloor \omega \,. \tag{B.20f}$$

B.9 Special Operators

Here we define some additional operators, which can be written as combinations of the already introduced functions. We also list some of the rules for these operators.

Codifferential

We define the codifferential $d^\dagger$ of the exterior derivative d by

$$d^\dagger \psi = (-1)^{n(p-1)+\mathrm{Ind}(g)} \; {}^\star d \, {}^\star \psi \,. \tag{B.21}$$

The codifferential decreases the form degree by 1. It maps p-forms onto $(p-1)$-forms.

The following two properties,

$$\text{the codifferential } d^\dagger \text{ is } \mathbb{R}\text{-linear ,} \tag{B.22a}$$

$$d^\dagger \, (d^\dagger \, \omega) = 0 \quad \text{(nilpotency)} \tag{B.22b}$$

can be easily derived from the corresponding rules (B.15a), (B.11a) and (B.15c) for the exterior product and the Hodge operator.

Wave Operator or d'Alembert Operator

The *wave* or *d'Alembert operator* is defined to be the anticommuting sum of exterior derivative and the codifferential:

$$\Box := dd^\dagger + d^\dagger d = -d\,{}^\star d\,{}^\star - {}^\star d\,{}^\star d \; . \tag{B.23}$$

The following commutation rules are valid:

$$d \, \Box = \Box \, d \; , \tag{B.24a}$$

$${}^\star \, \Box = \Box \, {}^\star \; , \tag{B.24b}$$

$$d^\dagger \, \Box = \Box \, d^\dagger \; . \tag{B.24c}$$

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